Phys 721 F'23 Problem Set 9 Solutions

1 a) $\mathcal{L} = \frac{1}{2}(\partial_\mu \phi)^2 - \frac{m^2}{2}\phi^2 + \bar{\Psi}(i\partial\!\!\!/ - M)\Psi - g\,\bar{\Psi}(a + ib\gamma^5)\Psi\,\phi$

Consider the process $\Psi(A) + \bar{\Psi}(B) \to \Psi(a) + \bar{\Psi}(b)$.

The Wick diagrams that contribute at $\mathcal{O}(g^2)$ must have:

a Ψ field uncontracted to annihilate $\Psi(A)$

a $\bar{\Psi}$ to annihilate $\bar{\Psi}(B)$

a $\bar{\Psi}$ to create $\Psi(a)$

a Ψ to create $\bar{\Psi}(b)$

All remaining fields must be contracted.

The interacting Hamiltonian is $H_I = \int d^3x\, g\, \bar{\Psi}(a+ib\gamma^5)\Psi\phi$.

Expanding the time-ordered exponential to $\mathcal{O}(g^2)$,

$$T\exp\left(-i\int_{-\infty}^{\infty} H_I\, dt\right) \supset \frac{(-ig)^2}{2!}\int d^4x_1\, d^4x_2\, :\bar{\Psi}(a+ib\gamma^5)\Psi\phi(x_1)\,\bar{\Psi}(a+ib\gamma^5)\Psi\phi(x_2):$$

The corresponding Wick diagram is 

The matrix element we are after gets two contributions at this order:

① $$2\times\langle a,b|\frac{(-ig)^2}{2!}\int d^4x_1\, d^4x_2\, :\bar{\Psi}(a+ib\gamma^5)\Psi(x_1)\,\bar{\Psi}(a+ib\gamma^5)\Psi(x_2):|A,B\rangle$$

$$\times\ \phi(x_1)\phi(x_2)$$

and

② $$2\times\langle a,b|\frac{(-ig)^2}{2!}\int d^4x_1\, d^4x_2\, :\bar{\Psi}(a+ib\gamma^5)\Psi(x_1)\,\bar{\Psi}(a+ib\gamma^5)\Psi(x_2):|A,B\rangle$$

$$\times\ \phi(x_1)\phi(x_2)$$

The factors of 2 come from the equivalent contributions with $x_1 \leftrightarrow x_2$.

The remainder of the calculation is identical to the calculation of $e^+ + e^- \to e^+ + e^-$ in the notes, except that A^μ is now ϕ, and γ^μ is now $(a + ib\gamma^5)$.

The contraction $\overline{\phi(x_1)\,\phi(x_2)}$ gives a factor $\dfrac{i}{(p_A+p_B)^2 - m^2 + i\epsilon}$

in the 1st term (labeled ① on the previous page), and a factor $\dfrac{i}{(p_a-p_A)^2 - m^2 + i\epsilon}$ in the 2nd term (labeled ②).

The net result for the two contributions are then,

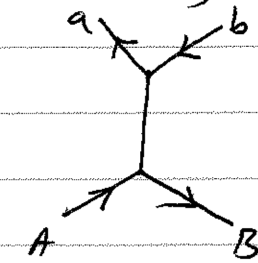
$$① = -(-ig)^2\, \bar{u}^{r_a}(p_a)(a+ib\gamma^5)v^{r_b}(p_b)\;\bar{v}^{r_B}(p_B)(a+ib\gamma^5)u^{r_A}(p_A)$$

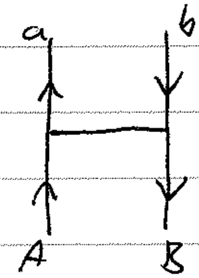
$$\times \frac{i}{(p_A+p_B)^2 - m^2 + i\epsilon}$$

$$② = (-ig)^2\, \bar{u}^{r_a}(p_a)(a+ib\gamma^5)u^{r_A}(p_A)\;\bar{v}^{r_B}(p_B)(a+ib\gamma^5)v^{r_b}(p_b)$$

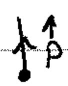
$$\times \frac{i}{(p_a-p_A)^2 - m^2 + i\epsilon}$$

The corresponding Feynman diagrams are:

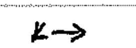
① = 

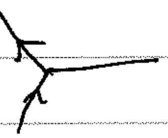
② = 

The Feynman rules are:

External lines:  $\bar{u}^r(p)$  $v^r(p)$

 $u^r(p)$  $\bar{v}^r(p)$

Propagator:  $\dfrac{i}{k^2-m^2+i\epsilon}$

Vertex:  $(a+ib\gamma^5)(-ig)$