Physics 721, Fall 2023
Problem Set 4
Due Monday, October 9.

1. *Hamiltonian and Momentum of the Complex Scalar Field*

Consider a complex scalar field $\phi(x)$ with Lagrangian density

$$\mathcal{L} = (\partial_\mu \phi)^\dagger (\partial^\mu \phi) - m^2 \phi^\dagger \phi.$$

a) Expand $\phi(x)$ in plane wave solutions to the equations of motion and derive an expression for the Hamiltonian in terms of creation and annihilation operators.

b) Similarly, derive an expression for the spatial momentum in terms of creation and annihilation operators.

c) Explain why both the particle and antiparticle creation operators create particles with positive energy.

2. *Propagators for the Complex Scalar Field*

Consider the free complex scalar field $\phi(x)$ from Problem 1.

a) Using only the equations of motion and the equal time commutation relations, show that for a complex scalar with mass m,

$$\left(\partial_\mu \partial^\mu + m^2\right) \langle 0| T\left[\phi(x)\phi^\dagger(y)\right] |0\rangle = -i\, \delta^4(x-y),$$

where the derivatives are with respect to the coordinates x, and T is the time ordering symbol. Recall that $\frac{\partial}{\partial t}\theta(t-t_0) = \delta(t-t_0)$.

Hence, $i\langle 0| T\left[\phi(x)\phi^\dagger(y)\right] |0\rangle$ is a Green's function for the Klein-Gordon operator.

b) Decompose $\phi(x)$ in plane wave solutions to the equations of motion, and use the harmonic oscillator commutation relations to calculate $\langle 0|T\left[\phi(x)\phi(y)\right]|0\rangle$, $\langle 0|T\left[\phi^\dagger(x)\phi^\dagger(y)\right]|0\rangle$, and $\langle 0|T\left[\phi(x)\phi^\dagger(y)\right]|0\rangle$. Compare with the Feynman propagator for the real scalar field.