Physics 721, Fall 2023
Final Exam
Due 12:00pm Monday, December 11, 2023
Submit your completed exam to my mailbox in Small 123 or by email to erlich@physics.wm.edu as a pdf.

You may consult any notes you have taken, any notes and homework solutions I have given you, and Peskin & Schroeder's textbook. That is all you may consult, out of fairness to your classmates. You should work alone. *Hint*: You will find the gamma matrix trace and contraction identities in Chapter 5.1 and Appendix A.3 of Peskin & Schroeder useful.

If you have any questions, feel free to stop by my office, email me at erlich@physics.wm.edu, or phone me on my cell phone at (757)272-2697.

You should not do more than six hours of work on the exam, and the exam must be completed over a contiguous twelve-hour period beginning when you first look at the problem.

1. $\mu \to e\gamma$

In this problem you will study the process $\mu \to e\gamma$, a process which does not occur in the Standard Model in the absence of neutrino masses.

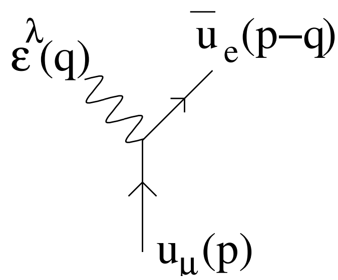
a) (10 points) The muon is identical to the electron except for its mass. What is the Lagrangian density for the theory of electrons and muons coupled to electromagnetism?

b) (30 points) What are the conserved charges due to the non-spacetime symmetries in this theory, and their physical interpretation? Express the charges in terms of the fields, and in terms of the particle creation and annihilation operators.

c) (10 points) Explain, based on symmetries, why the process $\mu \to e\gamma$ is forbidden in the theory described by your Lagrangian from part (a).

For the rest of this problem you should approximate the electrons as being massless.

d) (50 points) Neutrino masses can violate the symmetries that prohibit leptonic flavor-changing processes like $\mu \to e\gamma$. It can be argued based on Lorentz symmetry and gauge invariance that in the massless electron limit, the effective $\mu \to e\gamma$ vertex would have the form:



$$iM(\mu \to e\gamma) = A\,\varepsilon^{\lambda}(q)\,\overline{u}_e(p-q)\,\left(iq^{\nu}\,[\gamma_\lambda, \gamma_\nu]\,(1+\gamma_5)\right)\,u_\mu(p),$$

where A is some constant, u_e and u_μ are the Dirac spinors describing the electron and muon, and ε^λ is the photon polarization vector.

What is the decay rate of unpolarized muons into electrons and photons in the muon rest frame?