Phys 475 Problem 6 Solutions
5/10

6.3.10 $\vec{r}' = \vec{r} + a\vec{F}$ for some constant a.

$$\vec{r}' \times \vec{F} = (\vec{r} + a\vec{F}) \times \vec{F} = \vec{r} \times \vec{F} + a\,\vec{F} \times \vec{F} = \vec{r} \times \vec{F}$$
(because $\vec{F} \times \vec{F} = 0$)

$\vec{r}'' = \vec{r} + a\vec{n}$ for some constant a.

$$\vec{n} \cdot \vec{r} \times \vec{F} = \epsilon_{ijk}\, n_i\, r_j\, F_k$$
$$\vec{n} \cdot \vec{r}'' \times \vec{F} = \epsilon_{ijk}\, n_i (r_j + a n_j) F_k$$
$$= \epsilon_{ijk}\, n_i\, r_j\, F_k + a\, \epsilon_{ijk}\, n_i\, n_j\, F_k$$
$$= \vec{n} \cdot \vec{r} \times \vec{F} + 0$$
(because $\epsilon_{ijk}\, n_i\, n_j = 0$ by antisymmetry of ϵ_{ijk})

6.3.14 $\vec{A} \times (\vec{B} \times \vec{C}) + \vec{B} \times (\vec{C} \times \vec{A}) + \vec{C} \times (\vec{A} \times \vec{B})$

$$= \left(\cancel{(\vec{A}\cdot\vec{C})\vec{B}} - \cancel{(\vec{A}\cdot\vec{B})\vec{C}}\right) + \left(\cancel{(\vec{B}\cdot\vec{A})\vec{C}} - \cancel{(\vec{B}\cdot\vec{C})\vec{A}}\right) + \left(\cancel{(\vec{C}\cdot\vec{B})\vec{A}} - \cancel{(\vec{C}\cdot\vec{A})\vec{B}}\right)$$
$$= 0.$$

6.3.16 $\vec{L} = m\vec{r} \times (\vec{\omega} \times \vec{r})$

$$= m(\vec{r}\cdot\vec{r})\vec{\omega} - m(\vec{r}\cdot\vec{\omega})\vec{r}$$

If $\vec{r} \perp \vec{\omega}$ then $\vec{r}\cdot\vec{\omega} = 0$.

Then, $\vec{L} = m(\vec{r}\cdot\vec{r})\vec{\omega} = m r^2 \vec{\omega}$

$|\vec{L}| = m r^2 \omega$, where $\omega = |\vec{\omega}|$.

With $v = \omega r$, $\boxed{|\vec{L}| = mvr}$

6.4.6 $\vec{F} = q\,\vec{v} \times \vec{B}$. $\vec{v} \cdot \vec{F} = q\,\vec{v} \cdot \vec{v} \times \vec{B} = 0 \Rightarrow \boxed{\vec{v} \perp \vec{F}}$

$\frac{d}{dt}(\vec{v}\cdot\vec{v}) = 2\vec{v}\cdot\frac{d\vec{v}}{dt} = \frac{2}{m}\vec{v}\cdot\vec{F} = 0 \rightarrow \boxed{|\vec{v}| = \sqrt{\vec{v}\cdot\vec{v}} \text{ constant}}$

If $\vec{B} = B\hat{z}$, B constant, and $\vec{v}$ in (x,y) plane, then

$\vec{v} \perp \vec{B}$, $\boxed{|\vec{F}| = qvB = \text{constant}}$

6.6.17 $r = \sqrt{x^2+y^2}$.

$$\nabla r = \hat{\imath}\,\frac{\partial r}{\partial x} + \hat{\jmath}\,\frac{\partial r}{\partial y} = \hat{\imath}\cdot\frac{x}{\sqrt{x^2+y^2}} + \hat{\jmath}\,\frac{y}{\sqrt{x^2+y^2}}$$

$$= \boxed{\hat{\imath}\,\frac{x}{r} + \hat{\jmath}\,\frac{y}{r}}$$

In cylindrical coordinates,

$$\nabla r = \hat{e}_r\,\frac{\partial r}{\partial r} + \hat{e}_\theta\cdot\frac{1}{r}\,\underbrace{\frac{\partial r}{\partial\theta}}_{0}$$

$$= \hat{e}_r = \boxed{\hat{\imath}\cos\theta + \hat{\jmath}\sin\theta}$$

Using $x = r\cos\theta$, $y = r\sin\theta$, the boxed expressions are equal.

6.7.4 $\vec{V} = y\,\hat{\imath} + z\,\hat{\jmath} + x\,\hat{k}$

$$\nabla\cdot\vec{V} = \frac{\partial y}{\partial x} + \frac{\partial z}{\partial y} + \frac{\partial x}{\partial z} = \boxed{0}.$$

$$\nabla\times\vec{V} = \hat{\imath}\left(\frac{\partial x}{\partial y} - \frac{\partial z}{\partial z}\right) + \hat{\jmath}\left(\frac{\partial y}{\partial z} - \frac{\partial x}{\partial x}\right) + \hat{k}\left(\frac{\partial z}{\partial x} - \frac{\partial y}{\partial y}\right)$$

$$= \boxed{-\hat{\imath} - \hat{\jmath} - \hat{k}}$$

6.7.9 $$\nabla^2\left(x^3 - 3xy^2 + y^3\right) = \frac{\partial^2}{\partial x^2}\left(x^3 - 3xy^2 + y^3\right) + \frac{\partial^2}{\partial y^2}\left(x^3 - 3xy^2 + y^3\right) + \frac{\partial^2}{\partial z^2}\left(x^3 - 3xy^2 + y^3\right)$$

$$= \frac{\partial}{\partial x}\left(3x^2 - 3y^2\right) + \frac{\partial}{\partial y}\left(-6xy + 3y^2\right) + \frac{\partial}{\partial z}(0)$$

$$= 6x - 6x + 6y = \boxed{6y}$$