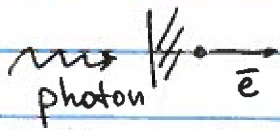
How to detect light?

Classical photodetection - based on photoelectric effect




Ideal case

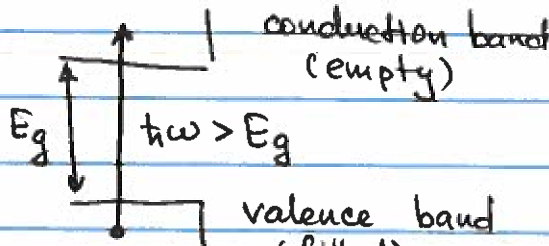
# photons = # photoelectrons

Photocurrent $I_{ph} = e \times (\text{photon flux}) = e \dfrac{P_{light}}{\hbar\omega}$

Quantum efficiency - the ratio b/w # photoelectron and incoming photons $\eta = N_e / N_{ph} \leq 1$

$$I_{ph} = \eta e \, P_{light} / \hbar\omega$$

Inside a photodetector (typically a semiconductor)




Photons, when absorbed, transfer electrons into the empty conduction band, where they can conduct, creating photocurrent

The bandgap of the material determines the spectral range of operation

Silicon : visible (0.4-0.9 μm) (can have very high quantum efficiency)

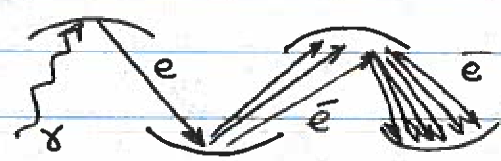
In-doped silicon: vis + near UV (0.2-0.8 μm)

Germanium : near-IR (1-2 μm)

"Strong" light — photocurrent propotional to the mean photoflux (mean photon number per unit time) $I_{ph} \propto \langle n_{photons} \rangle$

Can we use them to detect individual photons? Main problem is dark noise — ~~free~~ thermally induced ~~therm~~ free electrons even in the absence of light. That is why best detectors and cameras are often cooled.

Avalanch photodiodes (APD) or photomultipliers Can distinguish b/w no photons and a single / few photons



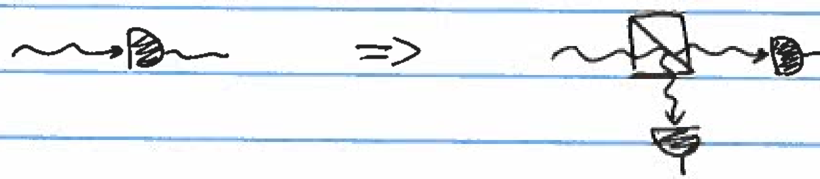

Initial photoelectron is accelerated by a high-V field toward the next plate. ~~Since~~ As more and more electrons are kicked at each plate, the avalanch is created — macroscopically measurable electric pulse

Drawbacks: – low quantum efficiency (~10% photomultipliers, ~50% APDs)

– vast variation in each avalanch amplitude => impossible to extract information about the initial # of photons

How can we measure photon statistics then?

Photo multiplexing



single photon – only one detector clicks!

two photons – 50% chance two detectors click

or

Photon-resolving detectors (expensive, exclusive)

What do we measure?

– Photon statistics $|\psi\rangle = \sum_{n=0}^{\infty} c_n |n\rangle$

probability to detect n-photons $p_n = |c_n|^2$

One can create a photo count hystogram to detect correct statistics

– Second-order correlation function

$$g^{(2)}(\tau) = \frac{\langle \hat{I}(t)\hat{I}(t+\tau)\rangle}{\langle I(t)\rangle^2} \Rightarrow \frac{\langle \hat{E}^+(t)\hat{E}^+(t+\tau)\hat{E}(t+\tau)\hat{E}(t)\rangle}{\langle \hat{E}^+\hat{E}\rangle^2}$$

or

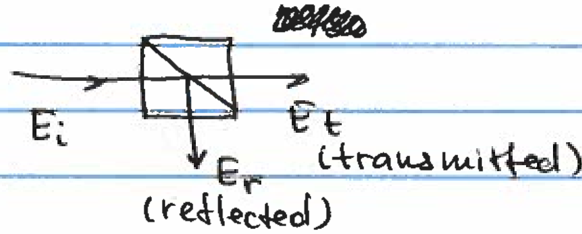
$$g^{(2)}(\tau) = \frac{\langle \hat{a}^+(t)\hat{a}^+(t+\tau)\hat{a}(t+\tau)\hat{a}(t)\rangle}{\langle \hat{a}^+\hat{a}\rangle^2}$$

$$g^{(2)}(0) = \frac{\langle \hat{n}(\hat{n}-1)\rangle}{\langle \hat{n}\rangle^2}$$

Single-photon source $g^{(2)}(0) = 0$

## Quantum beam splitter

Beam splitter is a device to divide a light beam into two by means of partial reflection and transmission




(reverse - beam combining

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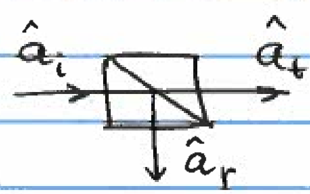
$E_t = tE_i$ , $E_R = rE_i$ $\qquad |t|^2+|r|^2 = 1$ energy conservation

Quantum case $\quad E \to \hat{a}$

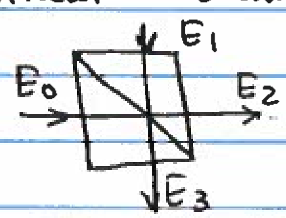



$\hat{a}_t = t\hat{a}_i$
$\hat{a}_r = r\hat{a}_i$ $\quad$ (?)

$[\hat{a}_t, \hat{a}_t^+] = [t\hat{a}_i, t^*\hat{a}_i^+] = |t|^2[\hat{a}_i, \hat{a}_i^+] = |t|^2 \leq 1$ (???)

Improper model! we forgot the vacuum

Linear beamsplitter - two inputs & two outputs




| | $E_1$ | $E_0$ |
|---|---|---|
| $E_2$ | $r$ | $t'$ |
| $E_3$ | $t$ | $r'$ |

The difference b/w $(r,t)$ and $(r',t')$ - possible phase difference

$E_2 = rE_1 + t'E_0$
$E_3 = tE_1 + r'E_0$

$$\begin{pmatrix} E_2 \\ E_3 \end{pmatrix} = \begin{pmatrix} t' & r \\ r' & t \end{pmatrix} \begin{pmatrix} E_0 \\ E_1 \end{pmatrix}$$

$E_i \to \hat{a}_i$

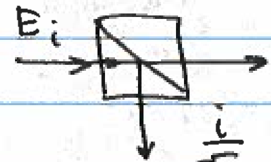
$$\begin{pmatrix} \hat{a}_2 \\ \hat{a}_3 \end{pmatrix} = \begin{pmatrix} t' & r \\ r' & t \end{pmatrix} \begin{pmatrix} \hat{a}_0 \\ \hat{a}_1 \end{pmatrix}$$

$$[\hat{a}_2, \hat{a}_2^+] = [t'\hat{a}_0 + r\hat{a}_1,\ t'^*\hat{a}_0^+ + r^*\hat{a}_1^+] =$$
$$= |t'|^2 [\hat{a}_0, \hat{a}_0^+] + |r|^2 [\hat{a}_1, \hat{a}_1^+] = |t'|^2 + |r|^2 = 1$$

always true when ~~[illegible]~~ $|t'| = |t|$, $|r'| = |r|$

Simple 50/50 beam splitter

Classical:




$\frac{1}{\sqrt{2}} E_i = E_t$

$\frac{i}{\sqrt{2}} E_i = E_r$ ( $\pi/2$ phase change)

Quantum: $\hat{a}_2 = \frac{1}{\sqrt{2}}(\hat{a}_0 + i\hat{a}_1)$ $\quad \hat{a}_3 = \frac{1}{\sqrt{2}}(i\hat{a}_0 + \hat{a}_1)$

We can write this as a unitary transformation
$$\begin{pmatrix} \hat{a}_2 \\ \hat{a}_3 \end{pmatrix} = \hat{U}^+ \begin{pmatrix} \hat{a}_0 \\ \hat{a}_1 \end{pmatrix} \hat{U}$$

$$\hat{U} = e^{i\pi/4 (\hat{a}_0^+ \hat{a}_1 + \hat{a}_0 \hat{a}_1^+)}$$

It is inconvenient to use Heisenberg representation: the states are invariant, the operators are modified. Instead, we will use Schrodinger picture: the operators are invariant, but the states are changing

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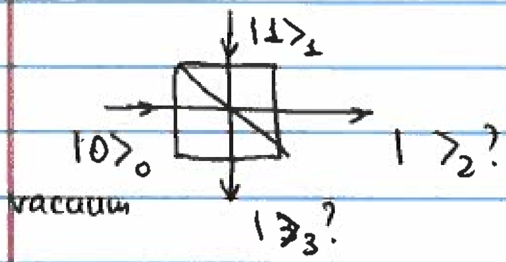
Handy expressions

$$\hat{a}_0 = \frac{1}{\sqrt{2}}(\hat{a}_2 - i\hat{a}_3) \qquad \hat{a}_0^+ = \frac{1}{\sqrt{2}}(\hat{a}_2^+ + i\hat{a}_3^+)$$

$$\hat{a}_1 = \frac{1}{\sqrt{2}}(-i\hat{a}_2 + \hat{a}_3) \qquad \hat{a}_1^+ = \frac{1}{\sqrt{2}}(i\hat{a}_2^+ + \hat{a}_3^+)$$

Simplest example: a single photon on a beam splitter




Trivial but crucial thought: vacuum is vacuum in any basis

$$|0\rangle_0|0\rangle_1 \xrightarrow{BS} |0\rangle_2|0\rangle_3$$

Initial state (one photon @ port 1):

$$|0\rangle_0|1\rangle_1 = \hat{a}_1^+ \underbrace{|0\rangle_0|0\rangle_1}$$

$$\frac{1}{\sqrt{2}}(i\hat{a}_2+\hat{a}_3)\,|0\rangle_2|0\rangle_3 = \frac{1}{\sqrt{2}}\left(i|1\rangle_2|0\rangle_3 + |0\rangle_2|1\rangle_3\right)$$

as expected 50% chance of detecting a single photon @ channel 2, and 50% chance detecting it @ channel 3

Coherent state: $|0\rangle_0|\alpha\rangle_1 = \hat{D}_1(\alpha)|0\rangle_0|0\rangle_1$

$$\hat{D}_1 = \exp(\alpha\hat{a}_1^+ - \alpha^*\hat{a}_1) \xrightarrow{BS} \exp\left(\frac{\alpha}{\sqrt{2}}(i\hat{a}_2^+ + \hat{a}_3^+) - \alpha^*(-i\hat{a}_2 + \hat{a}_3)\right)$$

$$= \exp\left(\frac{i\alpha}{\sqrt{2}}\hat{a}_2^+ + \frac{i\alpha^*}{\sqrt{2}}\hat{a}_2 + \frac{\alpha}{\sqrt{2}}\hat{a}_3^+ - \frac{\alpha^*}{\sqrt{2}}\hat{a}_3\right) =$$

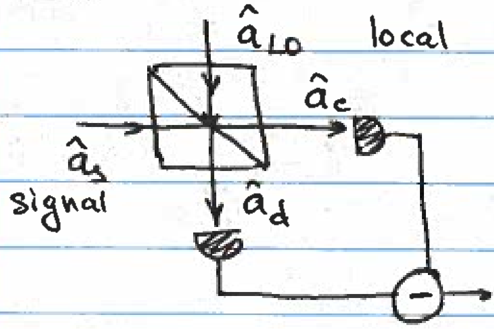
$$= \underbrace{\exp\left(\frac{i\alpha}{\sqrt{2}}\hat{a}_2^+ - \left(\frac{i\alpha}{\sqrt{2}}\right)^*\hat{a}_2\right)}_{\hat{D}_2\left(\frac{i\alpha}{\sqrt{2}}\right)} \underbrace{\exp\left(\frac{\alpha}{\sqrt{2}}\hat{a}_3^+ - \frac{\alpha^*}{\sqrt{2}}\hat{a}_3\right)}_{\hat{D}_3\left(\frac{\alpha}{\sqrt{2}}\right)}$$

Output state $\hat{D}_2\left(\frac{i\alpha}{\sqrt{2}}\right)\hat{D}_3\left(\frac{\alpha}{\sqrt{2}}\right)|0\rangle_2|0\rangle_3 = \left|\frac{i\alpha}{\sqrt{2}}\right\rangle_2\left|\frac{\alpha}{\sqrt{2}}\right\rangle_3$

A coherent state splits into two of equal amplitudes

## Balanced homodine detection

We can take advantage of high quantum efficiency of the conventional detectors, by cleverly "amplifying" a weak quantum signal.




local oscillator (LO)

Setup: mix the quantum signal with a strong coherent state (LO) on a 50/50 beam splitter, measure two photocurrents, analyze their difference.

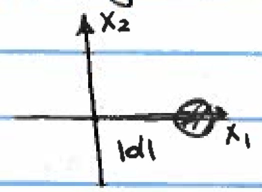
$$i_{c,d} \propto \langle \hat{n}_{c,d} \rangle \qquad \delta i \sim \langle \delta \hat{n} \rangle = \langle \hat{n}_c - \hat{n}_d \rangle$$

$$\hat{n}_c = \hat{a}_c^+ \hat{a}_c = \frac{1}{2} ( \hat{a}_s^+ - i \hat{a}_{LO}^+ )( \hat{a}_s + i \hat{a}_{LO} ) =$$
$$= \frac{1}{2} ( \hat{a}_{LO}^+ \hat{a}_{LO} + i \hat{a}_{LO} \hat{a}_s^+ - i \hat{a}_{LO}^+ \hat{a}_s + \hat{a}_s^+ \hat{a}_s )$$

$$\hat{n}_d = \hat{a}_d^+ \hat{a}_d = \frac{1}{2} ( -i \hat{a}_s^+ + \hat{a}_{LO}^+ )( i \hat{a}_s + \hat{a}_{LO} ) =$$
$$= \frac{1}{2} ( \hat{a}_{LO}^+ \hat{a}_{LO} + i \hat{a}_{LO} \hat{a}_s^+ + i \hat{a}_s \hat{a}_{LO}^+ + \hat{a}_s^+ \hat{a}_s )$$

$$\delta \hat{n} = i ( \hat{a}_s^+ a_{LO} - \hat{a}_s \hat{a}_{LO}^+ )$$

Normally, the local oscillator is a strong coherent state $|\alpha_{LO}\rangle$, $|\alpha_{LO}| \gg 1$




$$\langle \hat{a}_{LO} \rangle = \alpha_{LO} = |\alpha_{LO}|\, e^{i\chi}$$

It is often convenient to use

$$\hat{a}_{LO} = \alpha_{LO} + \delta\hat{a}_{LO}$$

such that $\langle \delta\hat{a}_{LO} \rangle = 0$

If we assume $\langle \hat{a}_s \rangle \ll \alpha_{LO}$, we can in fact neglect the fluctuation term and replace $\hat{a}_{LO} \to \alpha_{LO}$ $\quad \hat{a}_{LO}^{+} \to \alpha_{LO}^{*}$

$$\delta\hat{n} = i\left( \hat{a}_s^{+} |\alpha_{LO}| e^{i\chi} - \hat{a}_s^{*} |\alpha_{LO}| e^{-i\chi} \right) =$$

$$= |\alpha_{LO}| \underbrace{\left( \hat{a}_s e^{-i\theta} + \hat{a}_s^{+} e^{i\theta} \right)}_{2\hat{X}_\theta} \qquad \theta = \chi + \pi/2$$

$$\langle \delta\hat{n} \rangle = 2|\alpha_{LO}| \langle \hat{X}_\theta \rangle$$

Same as classical effect (radio!)

If we measure average power output of the photodetector

Power $\propto \langle \delta i^2 \rangle \propto \langle (\delta\hat{n})^2 \rangle \propto \langle X_\theta^2 \rangle$

Power fluctuations = (instantaneous power − mean power)

$= \langle \delta\hat{i}^2 \rangle - \langle \delta i \rangle^2 \propto \langle X_\theta^2 \rangle - \langle X_\theta \rangle^2 = (\Delta X_\theta)^2$

quadrature fluctuations

If our signal is vacuum or ~~sq~~ a weak coherent state

$$\& \ (\Delta X_\theta)^2 = \frac{1}{4}$$

$$(\Delta i)^2_{coh} \propto 4|\alpha_{LO}|^2 \cdot \frac{1}{4} = |\alpha_{LO}|^2$$

Squeezed vacuum ( assume $\langle \Delta X_1 \rangle^2 = \frac{1}{4} e^{-2r}$

$\langle \Delta X_2 \rangle^2 = \frac{1}{4} e^{2r}$

squeezed quadrature corresponds $\theta = 0$

$$(\Delta i)^2_{sqz} \propto 4|\alpha_{LO}|^2 \cdot \frac{1}{4} e^{-2r} = (\Delta i)^2_{coh} e^{-2r}$$

$$(\Delta i)^2_{anti\text{-}sqz} \propto 4|\alpha_{LO}|^2 \cdot \frac{1}{4} e^{2r} = (\Delta i)^2_{coh} e^{2r}$$