Jayne - Cummings Hamiltonian

$$\hat{H} = \frac{1}{2}\hbar\omega_0 \hat{\sigma}_z + \hbar\omega \hat{a}^+\hat{a} + \hbar g (\hat{\sigma}_- \hat{a}^+ + \hat{\sigma}_+ \hat{a})$$

For a given number of photons n

$$\hat{H}_n = \hbar\omega(n+\tfrac{1}{2})\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} + \frac{1}{2}\hbar \begin{pmatrix} \Delta & 2g\sqrt{n+1} \\ 2g\sqrt{n+1} & -\Delta \end{pmatrix}$$

$\underbrace{\hphantom{\hat{H}_n = \hbar\omega(n+\tfrac{1}{2})\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}}}$
overall energy shift
(neglectable)

Let's find the eigenstates of this Hamiltonian

$$\begin{vmatrix} \Delta - \lambda & 2g\sqrt{n+1} \\ 2g\sqrt{n+1} & -\Delta - \lambda \end{vmatrix} = 0$$

Generalized ~~Rabi~~ Rabi
frequency

$$-(\Delta^2 - \lambda^2) - 4g^2(n+1) = 0 \qquad \lambda_{1,2} = \pm\sqrt{\Delta^2 + 4g^2(n+1)} = \tilde{g}_n$$

Two eigen state energies $\hbar\omega(n+\tfrac{1}{2}) + \lambda_{1,2}$

$$E_{1n} = \hbar\omega(n+\tfrac{1}{2}) + \tfrac{1}{2}\hbar\tilde{g}_n \qquad E_{2n} = \hbar\omega(n+\tfrac{1}{2}) - \tfrac{1}{2}\hbar\tilde{g}_n$$

Corresponding dressed states

$$|1n\rangle = \sin\theta_n |e, n\rangle + \cos\theta_n |g, n+1\rangle$$
$$|2n\rangle = \cos\theta_n |e, n\rangle - \sin\theta_n |g, n+1\rangle$$

$$\cos\theta_n = \frac{\tilde{g}_n - \Delta}{\sqrt{(\tilde{g}_n - \Delta)^2 + 4g^2(n+1)}} \qquad \sin\theta_n = \frac{2g\sqrt{n+1}}{\sqrt{(\tilde{g}_n - \Delta)^2 + 4g^2(n+1)}}$$

For $\Delta = 0$ (resonant case)
$$E_{n_{1,2}} = \hbar\omega(n+\tfrac{1}{2}) \pm \hbar g\sqrt{n+1}$$

$$E_{1n} - E_{2n} = 2\hbar g\sqrt{n+1}$$
energy splitting b/w two dressed states

Even though the energy of a photon matches precisely the atomic resonance, the dressed states are shifted. That strongly affects the responce of the system on the consequtive photons.

Weak Coupling
energy levels for
photons - atom system

Strong coupling
(dressed state
picture)

$|a,3\rangle$ $|b,2\rangle$ $\Delta E_{n=2} = 2\sqrt{3}\,\hbar g$

$|a,2\rangle$ $|b,1\rangle$ $\Delta E_{n=1} = 2\sqrt{2}\,\hbar g$

$|a,1\rangle$ $|b,0\rangle$ $\Delta E_{n=0} = 2\hbar g$

$\hbar\omega_o$
$=\hbar\omega_0$

$|a,0\rangle$

$$\Delta E_n = 2\hbar g\sqrt{n+1} \longrightarrow \text{collective enhancement}$$
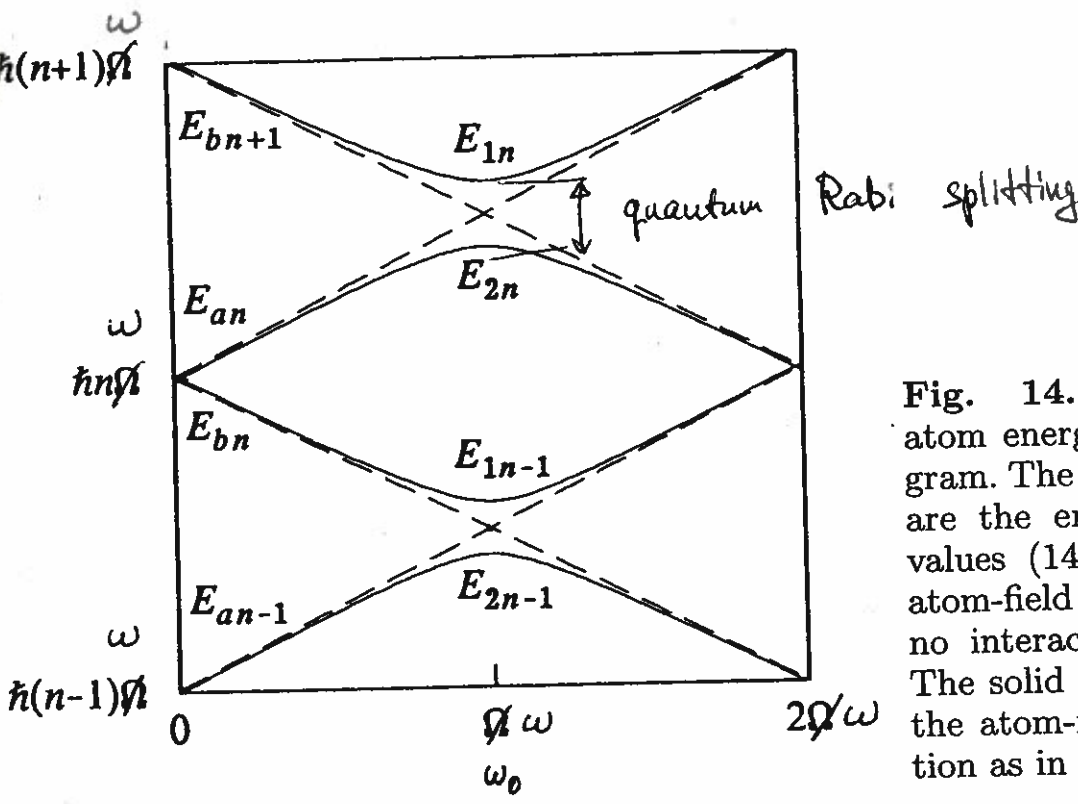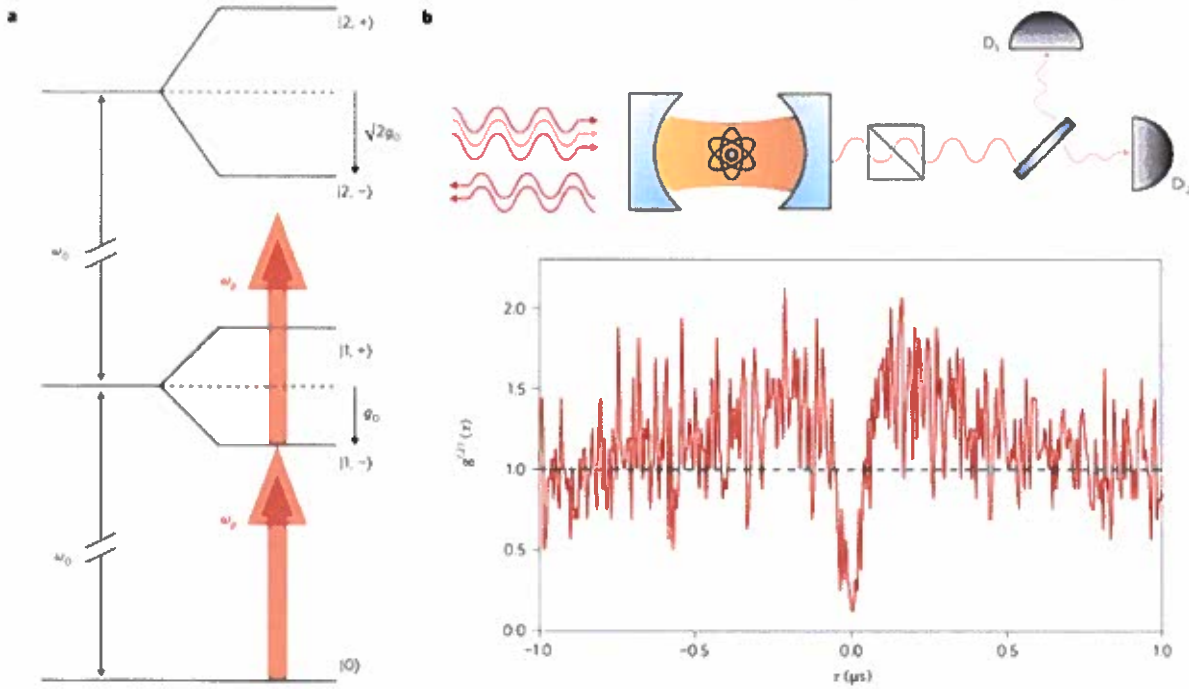
Fig. 14.1. Dressed atom energy level diagram. The dashed lines are the energy eigenvalues (14.11) for the atom-field system with no interaction energy. The solid lines include the atom-field interaction as in (14.14)