PHYS 690 Quantum and Nonlinear Optics

Problem set # 3 *(discussion date - November 3)*

P1 The expression for the electric field operator is given by:
$\hat{E} = \sqrt{\frac{\hbar\omega}{\epsilon_0 V}}(\hat{a}e^{-i\omega t} + \hat{a}^\dagger e^{i\omega t})\sin kz$
The front factor - $\sqrt{\frac{\hbar\omega}{\epsilon_0 V}}$ - is often referred to as an "electric field of a single photon", even though $\langle 1|\hat{E}|1\rangle = 0$. Explain why this still a reasonable name for this value.

P2 Prove that coherent states are not orthogonal, *i.e.* that
$\langle\alpha|\beta\rangle = exp\{-\frac{1}{2}(|\alpha|^2 + |\beta|^2 - 2\alpha^*\beta)\}$.

P3 A single photon source is a necessary component of many quantum cryptography devices. Due to the lack of commercially available true single photon sources, many experiments use a strongly attenuated laser pulses with average photon number per pulse much less than one. This is not optimal solution, since there is always a non-zero probability for having a pair of photons, which is a security risk. Assume that for a particular protocol sets a limit $p \ll 1$ to the ratio of two-photon pulses with respect to the singe-photon pulses. What is the average number of photons in this attenuated coherent state?

P4 A Schrodinger cat state is a superposition of two coherent states: $|\psi\rangle \simeq (|\alpha\rangle + |-\alpha\rangle)/\sqrt{2}$. Calculate average electric field of this state, and average photon number. What do these answers imply?

P5 A photon-added coherent state is the state $|\alpha, 1\rangle = \mathcal{N}\hat{a}^\dagger|\alpha\rangle$. Find the normalization factor $\mathcal{N}$ of this state, and determine its photon statistics.

P6 Consider the superposition of the vacuum and 10 photon number state: $|\psi\rangle \simeq (|0\rangle + |10\rangle)/\sqrt{2}$. Calculate the average photon number. Next assume that a single photon is absorbed and recalculate the average photon number. Does your result seem sensible? Can you resolve a possible controversy?

P7 The experimenters reported 5 dB intensity squeezed vacuum. Help theorists to write the squeezing operator to produce that amount of squeezing. How one must change it to obtain 5 dB of phase squeezed vacuum?

P8 Calculate the variances of the quadrature operators $\Delta X_{1,2}$ in a coherent state $|\alpha\rangle$ and show that they are the same as for the vacuum state.