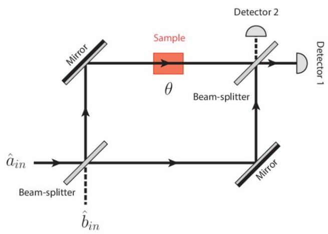
PHYS 690 Midterm test #2 (due 11.59 pm November 29)

You are allowed to use any of the textbooks and notes, recommended for this class, and your class materials; however, any additional resources are prohibited. One can use a computer for accessing the electronic textbook versions or carrying out numerical calculations (if necessary), but you are not allowed to search for the solutions on-line or to communicate with other people.



Problem 1: Squeezed-vacuum enhanced interferometer (40 points)
Consider a Mach-Zender interferometer and assume that one input mode is in a coherent state and that the other is in a squeezed vacuum state. Show that the phase fluctuations at the output can be reduced below the standard quantum limit ($\Delta\phi_{SQL} = 1/\sqrt{\bar{n}}$) to $\Delta\phi = e^{-r}/\sqrt{\bar{n}}$ and identify the squeezing quadrature that will provide this result. *Hint:* it may be easier to identify the relations between the creation and annihilation operators of two input and two output interferometer states, so that you don't have to transform the states inside the interferometer.

Problem 2: NOON state preparation (40 points)
In class we discuss the potential advantages of using "NOON" states for quantum enhanced measurements. Suppose you are tasked with a charge of producing such a state with $N = 3$ photons, shared between two outputs of a beam splitter. You have two options: send all three identical photons on one input of the beam splitter or send two photons in one input port, and the third one in the second input port. In either case, the desired state (i.e. all three photons emerging in one of the output) will be generated only in fraction of the trials.
(a) Calculate the probability to detect a NOON state $\left(|3\rangle|0\rangle + e^{i\phi}|0\rangle|3\rangle\right)/\sqrt{2}$ in both cases. Determine the value of the phase ϕ. In which case the rate of success is higher?
(b) Now let's consider a scenario that you have only single-photon sources, but you have access to multiple beam splitters. Construct a possible experimental arrangement to realize the $N = 3$ NOON state discussed above, indicating what kind of state selection is necessary at which stage.

Problem 3: Photon pair generation eigenstates(20 points)
Consider a pair state $|\eta\rangle$, which is defined as simultaneous eigenstate of the pair annihilation operator $\hat{a}\hat{b}$ with the eigenstate η (a complex number) and of the number difference operator $\hat{a}^\dagger\hat{a} - \hat{b}^\dagger\hat{b}$ with zero eigenvalue. Find the description of the state $|\eta\rangle$ in terms of the number states, and examine its properties.