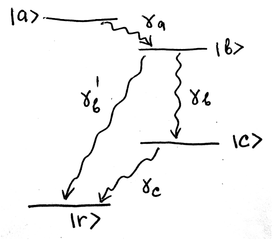
PHYS 690 Midterm 1 (due 12pm October 20, 2021)

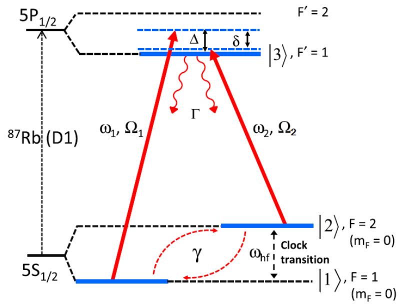
You are allowed to use any of the textbooks and notes, recommended for this class, and your class materials; however, any additional resources are prohibited. One can use a computer for accessing the electronic textbook versions or carrying out numerical calculations (if necessary), but you are not allowed to search for the solutions on-line or to communicate with other people.

Problem 1: Population inversion in a four-level scheme (40 points)



Consider a four-level system shown. The reservoir level r is the ground-state. Upper level a decays into level b with the decay rate γ_a, level b decays into the level c with the rate γ_b and to the reservoir level r with rate γ_b'. Finally, the lower level c decays into the ground state with the rate γ_c. An electric discharge effectively repopulates the state a at rate λ.
(a) Write down the equations for the steady-state **populations** of the levels b and c in the absence of any external optical fields.
(b) Assuming the atomic populations are fixed to the values you have calculated in part *(a)*, find the expression for susceptibility $\chi(\Delta)$ for an optical field, nearly resonant with the $|b\rangle \to |c\rangle$ transition. Here Δ is the frequency difference between the optical field and the transition frequency ω_{bc}.
(c) Show that depending on the values of decays and the repumping rate, it is possible to realize amplification for such an optical field. Find these conditions.

Problem 2: EIT-based atomic clocks (40 points)



In class we mostly considered the electromagnetically induced transparency (EIT) in which one of the optical fields (pump) was significantly stronger than the other (probe). This configuration is mostly relevant for quantum information applications for manipulating quantum signals. However, same effect can also be used for atomic clocks and magnetometers that relies on precision measurement of the hyperfine frequency in alkali metal atoms. In this case, however, the intensities of the two optical fields are typically equal, and the total transmission is measured. An example of the EIT-based atomic clock interrogation scheme is shown here. Two hyperfine states of a Rb atom $5S_{1/2}F = 1, 2$ form the basis of the Λ system, and $5P_{1/2}F = 1$ is the third, excited, state (we will disregard any effect of $5P_{1/2}F = 2$). Δ is a one-photon detuning of one of the optical field from the resonance (Ω_1 in the figure), and δ is the two-photon detuning. We assume that the absolute values of the Rabi frequencues of the two optical fields are the same, i.e., $|\Omega_1|^2 = |\Omega_2|^2$. The excited state population decays into both ground-state levels with the equal decay rates Γ, and there is a collisional decoherence between two ground states at a rate $\gamma \ll \Gamma$ (no population exchange between the two ground states).
(a) Does a dark state exist in this system? If yes, provide the corresponding wave function $|D\rangle$.
(b) Write down the Block equations for this three-level system, using the parameters denoted in the figure.
(c) Assuming the resonant case $\Delta = 0$ find total transmission through the system as a function of δ in a steady-state case. Clearly identify all the assumptions and simplifications you make in your calculations. *Hint 1: you can assume that populations of all levels are given by the dark state.*
Hint 2: for total transmission calculate the absorption of each optical field individually, then calculate changes in their total intensity.
(d) What are the refractive indices of each optical field as a function of δ?

Problem 3: Nonlinear frequency generation (20 points)

Calculate all wavelengths that can be generated in a $\chi^{(3)}$ nonlinear medium by a combination of 632.8 nm and 388 nm laser light.