Problem 6

After 90/10 beamsplitter, the transmitted quadratures are $\hat{X}_1^{(tr)} = t\hat{X}_1^{(sq)} + ir\,\hat{X}_2^{(vac)}$

$$\langle \hat{X}_1^2 \rangle = |t|^2 \langle \hat{X}_1^{(sq)2} \rangle + |r|^2 \langle \hat{X}_2^{(vac)2} \rangle =$$

$$= 0.9 \cdot \cancel{\tfrac{1}{4}} \langle \hat{X}_1^{(sq)2} \rangle + 0.1 \cdot \langle \hat{X}_{vac}^2 \rangle$$

Measured squeezing $\dfrac{\langle \hat{X}_1^2 \rangle}{\langle X_{vac}^2 \rangle} = 0.9 \cdot \dfrac{\langle X_1^{(sq)2} \rangle}{\langle X_{vac}^2 \rangle} + 0.1$

To work in dB: $Sq_{out}[dB] = 10 \cdot \log\left[|t|^2 \, 10^{\frac{Sq_{in}}{10}} + |r|^2\right]$

For $Sq_{in} = -5\,dB$, $Sq_{out} = -4.14\ dB$

Problem 7

In the homodyne detection
$\hat{i} \propto \delta\hat{n} = i(\hat{a}_s^+ \hat{a}_{LO} - \hat{a}_{LO}^+ \hat{a}_s)$

Presenting $\hat{a}_{LO} = d_{LO} + \delta\hat{a}_{LO}$, such that $\langle \delta\hat{a}_{LO} \rangle = 0$

$$\delta\hat{n} = i\left(\hat{a}_s^+ (d_{LO} + \delta\hat{a}_{LO}) - \hat{a}_s (d_{LO}^* + \delta\hat{a}_{LO}^+)\right) =$$

$$= i\underbrace{\left(d_{LO}\hat{a}_s^+ - d_{LO}^*\hat{a}_s\right)}_{2|d_{LO}|\hat{X}_\theta} + i\left(\hat{a}_s^+ \delta\hat{a}_{LO} - \hat{a}_s \delta\hat{a}_{LO}^+\right)$$

$\langle \delta n \rangle = 0$ since $\langle \hat{X}_\theta \rangle = 0$ and $\langle \delta\hat{a}_{LO} \rangle = 0$

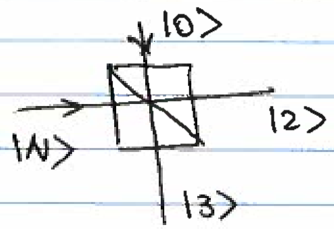
What we are interested in are fluctuations

$$\langle \delta n^2 \rangle = 4|d_{LO}|^2 \langle X_\theta^2 \rangle + |d_{LO}|\underbrace{\left(\langle \hat{a}_s^{+2} \rangle \langle \delta\hat{a}_{LO} \rangle + \text{similar terms}\right)}_{=0 \text{ since } \langle \delta a_{LO} \rangle = 0}$$

$$+ \underbrace{\langle \delta a_{LO}^2 \rangle \left(\langle a_s^+ \rangle^2 + \text{similar terms}\right)}$$

Since $\langle \delta a_{LO}^2 \rangle \ll |d_{LO}|^2$ these terms give negligible contributions

Problem 8



Initial state: $|N\rangle|0\rangle = \frac{1}{\sqrt{N!}}(\hat{a}_0^+)^N |0\rangle_0|0\rangle_1$

Final output

$$|\psi\rangle_{out} = \frac{1}{\sqrt{N!}}\left\{\frac{1}{2^{N/2}}(\hat{a}_2^+ + i\hat{a}_3^+)^N |0\rangle_2|0\rangle_3 = \right.$$

$$= \frac{1}{\sqrt{N!}}\frac{1}{2^{N/2}}\sum_{m=0}^{N}\frac{N!}{(N-m)!\,m!}(\hat{a}_2^+)^m(i\hat{a}_3^+)^{N-m}\quad|0\rangle_2|0\rangle_3$$

Only the "edge" terms create NOON state $(m=0 \;\&\; m=N)$

$$|\psi\rangle_{out} = \underbrace{\frac{1}{2^{N/2}}\left\{|N\rangle_2|0\rangle_3 + (i)^N|0\rangle_2|N\rangle_3\right\}}_{\frac{1}{2^{N/2-1/2}}\times \text{NOON state}} + \ldots\sum_{m=1}^{N-1}\ldots$$

Probability to obtain the NOON state

in $P_{NOON} = \left(\frac{1}{2^{(N-1)/2}}\right)^2 = \frac{1}{2^{N-1}}$

exponential scaling, very unfavorable!

HW 4 (solutions)

Problem 1

$$g^{(2)}(\tau) = \frac{\langle \hat{a}^+(t)\,\hat{a}^+(t+\tau)\,\hat{a}(t+\tau)\,\hat{a}(t)\rangle}{\langle \hat{a}^+(t)\hat{a}(t)\rangle^2}$$

$$g^{(2)}(0) = \frac{\langle \hat{a}^+\hat{a}^+\hat{a}\hat{a}\rangle}{\langle \hat{n}\rangle^2} = \frac{\langle \hat{a}^+(\hat{a}\hat{a}^+ - 1)\hat{a}\rangle}{\langle \hat{n}\rangle^2} =$$

$$= \frac{\langle \hat{n}^2 - \hat{n}\rangle}{\langle n\rangle^2}$$

A number state $\langle n|\hat{n}^2 - \hat{n}|n\rangle = n^2 - n$

$$g^{(2)}(0) = \frac{n^2-n}{n^2} = 1 - \frac{1}{n}$$

for $n=1$ $g^{(2)}(0) = 0$

Coherent state $\langle \alpha|\hat{a}^+\hat{a}^+\hat{a}\hat{a}|\alpha\rangle = |\alpha|^4$ $\quad g^{(2)}(0) = 1$

$\langle \alpha|\hat{a}^+\hat{a}|\alpha\rangle = |\alpha|^2$

Problem 2

Input state $|0\rangle_0|2\rangle_1 = \frac{1}{\sqrt{2}}(a_1^+)^2|0\rangle_0|0\rangle_1$

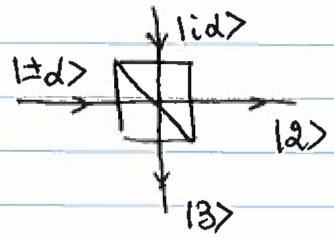
Output state $\frac{1}{\sqrt{2}} \times \left(\frac{1}{\sqrt{2}}(\hat{a}_3^+ + i\hat{a}_2^+)\right)^2 |0\rangle_2|0\rangle_3 =$

$$= \frac{1}{2\sqrt{2}}\left(\hat{a}_3^{+2} - \hat{a}_2^{+2} + 2i\hat{a}_3^+\hat{a}_2^+\right) = \underbrace{\frac{1}{2}\left(|0\rangle_2|2\rangle_3 - |2\rangle_2|0\rangle_3\right)}_{\frac{1}{\sqrt{2}} \times \text{NOON state } N=2} +$$

$$+ \frac{i}{\sqrt{2}}|1\rangle_2|1\rangle_3$$

3 Problem 3

$$|\alpha\rangle = e^{\alpha \hat a - \alpha^* \hat a^+} |0\rangle$$



Initial $|\alpha\rangle_0 |i\alpha\rangle_1 =$

$$= e^{\alpha \hat a_0 - \alpha^* \hat a_0^+} e^{i\alpha \hat a_1 + i\alpha^* \hat a_1^+} |0\rangle_0 |0\rangle_1$$

Output state : $\exp\left\{ \frac{\alpha}{\sqrt{2}} (\hat a_2 \cancel{+ i \hat a_3}) - \frac{\alpha^*}{\sqrt{2}} (\hat a_2^+ \cancel{+ i \hat a_3^+}) + \right.$

$$\left. + i\alpha/\sqrt{2} (\cancel{\hat a_3} - i \hat a_2) + i \frac{\alpha^*}{\sqrt{2}} (\cancel{\hat a_3^+} + i \hat a_2^+) \right\} |0\rangle_2 |0\rangle_3$$

$$= \exp\left\{ \sqrt{2} (\alpha \hat a_2 - \alpha^* \hat a_2^+) \right\} |0\rangle_2 |0\rangle_3 = |\sqrt{2}\alpha\rangle_2 |0\rangle_3$$

All light emerges from output $|2\rangle$

Similarly, if $|-\alpha\rangle$ is in the input $|0\rangle$, all the light will emerge from output $|3\rangle$

Clear differentiation of the input states

Problem 4

$$\begin{pmatrix}\hat{a}_2\\ \hat{a}_3\end{pmatrix} = \hat{U}_\theta^\dagger \begin{pmatrix}\hat{a}_0\\ \hat{a}_1\end{pmatrix}\hat{U}_\theta$$

$$\hat{a}_2 = \hat{U}_\theta^\dagger \hat{a}_0 \hat{U}_\theta = \left(e^{-i\frac{\theta}{2}(\hat{a}_0^\dagger \hat{a}_1 + \hat{a}_1^\dagger \hat{a}_0)}\right)^\dagger \hat{a}_0 \left(e^{-i\frac{\theta}{2}(\hat{a}_0^\dagger \hat{a}_1 + \hat{a}_1^\dagger \hat{a}_0)}\right)$$

$$= \hat{a}_0 - \frac{i\theta}{2}\left[(\hat{a}_0^\dagger \hat{a}_1 + \hat{a}_0 \hat{a}_1^\dagger), \hat{a}_0\right] + \frac{1}{2!}\left(\frac{i\theta}{2}\right)^2 \left[(\hat{a}_0^\dagger \hat{a}_1 + \hat{a}_1 \hat{a}_0), [\,]\right] + \dots$$

$$[\hat{a}_0^\dagger \hat{a}_1 + \hat{a}_0 \hat{a}_1^\dagger, \hat{a}_0] = -\hat{a}_1$$
$$[\hat{a}_0^\dagger \hat{a}_1 + \hat{a}_0 \hat{a}_1^\dagger, \hat{a}_1] = -\hat{a}_0$$

So each consecutive ~~operator~~ commutator in the series oscillates b/w $\hat{a}_0$ and $\hat{a}_1$

$$\hat{a}_2 = \hat{a}_0 + \frac{i\theta}{2}\hat{a}_1 + \frac{1}{2!}\left(\frac{i\theta}{2}\right)^2 \hat{a}_0 + \dots = i\sin\theta/2\, \hat{a}_1 + \cos\frac{\theta}{2}\hat{a}_0$$

Similarly, $\hat{a}_3 = \cos\frac{\theta}{2}\hat{a}_1 + i\sin\frac{\theta}{2}\hat{a}_0$

$$\begin{pmatrix}\hat{a}_2\\ \hat{a}_3\end{pmatrix} = \begin{pmatrix}\cos\theta/2 & i\sin\frac{\theta}{2}\\ i\sin\theta/2 & \cos\theta/2\end{pmatrix}\begin{pmatrix}\hat{a}_0\\ \hat{a}_1\end{pmatrix}$$

$r = i\sin\theta/2$
$t = \cos\theta/2$

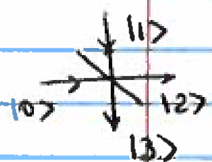
For $\theta = \pi/2$ $\quad |r| = |t| = 1/\sqrt{2}$ $\quad$ 50/50 Beamsplitter

Problem 5

For each input field we have two quadratures $\hat{X}_1^{(i)} = \frac{1}{2}(\hat{a} + \hat{a}^+)$

$$X_2^{(i)} = \frac{1}{2i}(\hat{a} - \hat{a}^+)$$

Beamsplitter transformation:



$$\hat{a}_2 = \frac{1}{\sqrt{2}}(\hat{a}_0 + i\hat{a}_1) \quad , \quad \hat{a}_2^+ = \frac{1}{\sqrt{2}}(\hat{a}_0^+ - i\hat{a}_1^+)$$

$$\hat{X}_1^{(2)} = \frac{1}{2}(\hat{a}_2 + \hat{a}_2^+) = \frac{1}{\sqrt{2}}\left[\frac{1}{2}(\hat{a}_0 + \hat{a}_0^+) + \frac{i}{2}(\hat{a}_1 - \hat{a}_1^+)\right] =$$

$$= \frac{1}{\sqrt{2}}\hat{X}_1^{(0)} - \frac{1}{\sqrt{2}}\hat{X}_2^{(1)} \quad ; \quad \hat{X}_2^{(2)} = \frac{i}{\sqrt{2}}\hat{X}_2^{(0)} - \frac{i}{\sqrt{2}}\hat{X}_1^{(1)}$$

$$\hat{a}_3 = \frac{1}{\sqrt{2}}(\hat{a}_1 + i\hat{a}_0) \qquad \hat{a}_3^+ = \frac{1}{\sqrt{2}}(\hat{a}_1 - i\hat{a}_0)$$

$$\hat{X}_1^{(3)} = \frac{1}{\sqrt{2}}\hat{X}_1^{(1)} - \frac{1}{\sqrt{2}}\hat{X}_2^{(0)} \; ; \; \hat{X}_2^{(3)} = \frac{i}{\sqrt{2}}\hat{X}_2^{(1)} - \frac{i}{\sqrt{2}}X_1^{(0)}$$

Non-50/50 beamsplitter

$$\hat{a}_2 = t\hat{a}_0 + r\hat{a}_1 \qquad \hat{a}_2^+ = t^*\hat{a}_0^+ + r^*\hat{a}_1^+$$

$$\hat{X}_1^{(2)} = \frac{1}{2}(\hat{a}_2 + \hat{a}_2^+) = \frac{1}{2}(t\hat{a}_0 + r\hat{a}_1) + \frac{1}{2}(t^*\hat{a}_0^+ + r^*\hat{a}_1^+) =$$

= { assume $t = t^*$, $r = -r^*$, like in problem 4 }

$$= \frac{1}{2}t(\hat{a}_0 + \hat{a}_0^+) + \frac{1}{2}r(\hat{a}_1 - \hat{a}_1^+) = t\hat{X}_1^{(0)} + ir\hat{X}_2^{(1)}$$