Parity non-conservation

Conservation of parity has been taken for granted for a long time.
Indeed, EM interaction is parity-conserving

$$\hat{H}_{el} = -e\vec{r}\cdot\vec{E} \quad \text{odd}\cdot\text{odd} = \text{even}$$

$$\hat{H}_{magn} = -\frac{e}{2mc}(\vec{L}+2\vec{S})\cdot\vec{B} \quad \text{even}\cdot\text{even} = \text{even}$$

Strong interaction is also parity-conserving
Weak interaction turned out to be non-parity-conserving!

History: two particle decays were recorded using cosmic rays

$$\theta^+ \longrightarrow \pi^+ + \pi^0 \quad \text{and} \quad \tau^+ \longrightarrow \pi^+ + \pi^+ + \pi^-$$

π's have negative parity $\Rightarrow$
θ^+ must be even and τ^+ must be odd
Turned out $\theta^+ = \tau^+ = K^+$, and parity is not conserved in its decay

First people who suggested that the weak interaction does not conserve parity were Lee and Yang

$$^{60}Co \longrightarrow {}^{60}Ni + e^- + \bar{\nu}_e$$

From experiment $\quad H \propto \langle\vec{S}\rangle\cdot\vec{p}$

nuclear spin

momentum of $\bar{e}$

one unit and no change of parity, it can be given only by the Gamow-Teller interaction. This is almost imperative for this experiment. The thickness of the radioactive layer used was about 0.002 inch and contained a few microcuries of activity. Upon demagnetization, the magnet is opened and a vertical solenoid is raised around the lower part of the cryostat. The whole process takes about 20 sec. The beta and gamma counting is then started. The beta pulses are analyzed on a 10-channel pulse-height analyzer with a counting interval of 1 minute, and a recording interval of about 40 seconds. The two gamma counters are biased to accept only the pulses from the photopeaks in order to discriminate against pulses from Compton scattering.

A large beta asymmetry was observed. In Fig. 2 we have plotted the gamma anisotropy and beta asymmetry *vs* time for polarizing field pointing up and pointing down. The time for disappearance of the beta asymmetry coincides well with that of gamma anisotropy. The warm-up time is generally about 6 minutes, and the warm counting rates are independent of the field direction. The observed beta asymmetry does not change sign with reversal of the direction of the demagnetization field, indicating that it is not caused by remanent magnetization in the sample.

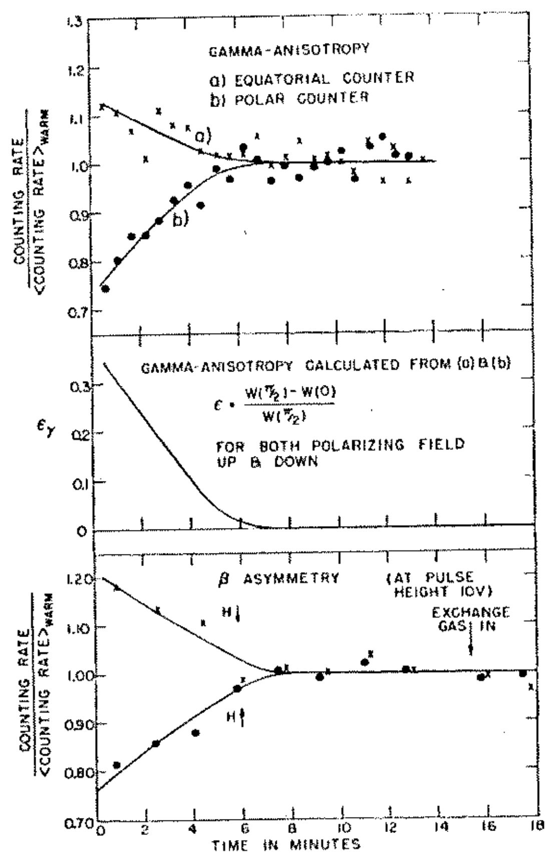

FIG. 2. Gamma anisotropy and beta asymmetry for polarizing field pointing up and pointing down.

The sign of the asymmetry coefficient, α, is negative, that is, the emission of beta particles is more favored in the direction opposite to that of the nuclear spin. This naturally implies that the sign for C_T and C_T' (parity conserved and parity not conserved) must be opposite. The exact evaluation of α is difficult because of the many effects involved. The lower limit of α can be estimated roughly, however, from the observed value of asymmetry corrected for backscattering. At velocity $v/c\approx 0.6$, the value of α is about 0.4. The value of $\langle I_z\rangle/I$ can be calculated from the observed anisotropy of the gamma radiation to be about 0.6. These two quantities give the lower limit of the asymmetry parameter $\beta(\alpha=\beta\langle I_z\rangle/I)$ approximately equal to 0.7. In order to evaluate α accurately, many supplementary experiments must be carried out to determine the various correction factors. It is estimated here only to show the large asymmetry effect. According to Lee and Yang[3] the present experiment indicates not only that conservation of parity is violated but also that invariance under charge conjugation is violated.[4] Furthermore, the invariance under time reversal can also be decided from the momentum dependence of the asymmetry parameter β. This effect will be studied later.

The double nitrate cooling salt has a highly anisotropic g value. If the symmetry axis of a crystal is not set parallel to the polarizing field, a small magnetic field will be produced perpendicular to the latter. To check whether the beta asymmetry could be caused by such a magnetic field distortion, we allowed a drop of $CoCl_2$ solution to dry on a thin plastic disk and cemented the disk to the bottom of the same housing. In this way the cobalt nuclei should not be cooled sufficiently to produce an appreciable nuclear polarization, whereas the housing will behave as before. The large beta asymmetry was not observed. Furthermore, to investigate possible internal magnetic effects on the paths of the electrons as they find their way to the surface of the crystal, we prepared another source by rubbing $CoCl_2$ solution on the surface of the cooling salt until a reasonable amount of the crystal was dissolved. We then allowed the solution to dry. No beta asymmetry was observed with this specimen.

More rigorous experimental checks are being initiated, but in view of the important implications of these observations, we report them now in the hope that they may stimulate and encourage further experimental investigations on the parity question in either beta or hyperon and meson decays.

The inspiring discussions held with Professor T. D. Lee and Professor C. N. Yang by one of us (C. S. Wu) are gratefully acknowledged.

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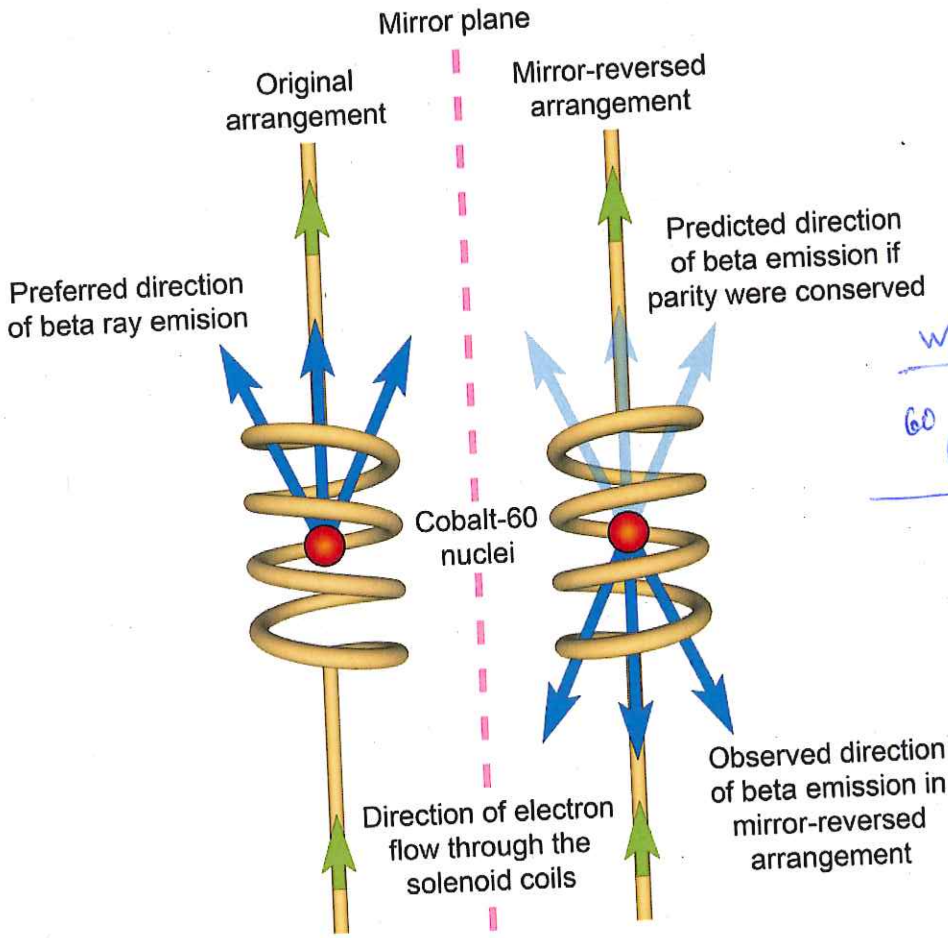
[1] T. D. Lee and C. N. Yang, Phys. Rev. 104, 254 (1956).
[2] Ambler, Grace, Halban, Kurti, Durand, and Johnson, Phil. Mag. 44, 216 (1953).
[3] Lee, Oehme, and Yang, Phys. Rev. (to be published).



weak interaction

$$^{60}Co \rightarrow {}^{60}Ni^{*} + \bar{e} + \bar{\nu}_e$$

$$Ni^{*} \rightarrow \gamma + Ni$$

EM - interaction conserves parity

$\vec{P} \to -\vec{P}$, $\vec{L} \to \vec{L}$

$\vec{P} \to \vec{P}$, $\vec{L} \to \vec{L}$, $q \to -q$

$\vec{P} \to -\vec{P}$, $\vec{L} \to \vec{L}$, $q \to -q$

$\mu^+ \to e^+ + \nu_e + \bar{\nu}_\mu$

helicity = angular momentum

helicity $\nu_e = +1$

helicity $\bar{\nu}_\mu = -1$

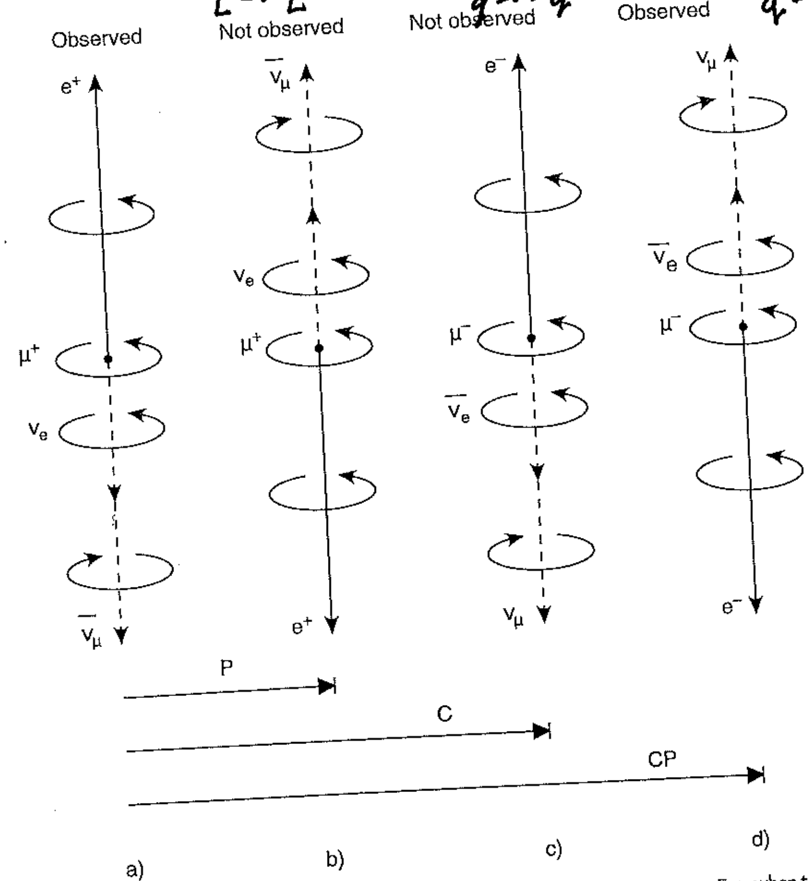


Figure 24.9 Diagrams illustrating linear momenta and spins of the decay products in the decays $\mu^+ \to e^+\nu_e\bar{\nu}_\mu$, $\mu^- \to e^-\bar{\nu}_e\nu_\mu$, when the positron or electron energy is near its maximum value. Cases a) and d) are observed, cases b) and c) are never observed.

was discovered in 1964 and will be discussed later, *CP* invariance does break down in other weak decays.

24.6 The *V-A* Law

The discovery of parity violation resulted in vigorous experimental activity that culminated within less than two years in a generalization of Fermi's theory, called the *V-A law*. This was enunciated independently by Sudarshan and Marshak, Feynman and Gell Mann, Sakurai, and Gershtein and Zeldovitch in 1958. Here it was proposed that the Lagrangian density for charged weak interactions (these were the only type known at the time) takes the following form.

$$\mathcal{L}_W = -\frac{G_F}{\sqrt{2}} J_\sigma^\dagger J^\sigma \tag{24.17}$$

Parity non-conservation in atoms is due to weak interaction b/w nucleous and an electron via exchange of Z-boson

$$\hat{H}_{weak} = \frac{G_F}{\sqrt{2}} \frac{1}{2mc\hbar} Q_W \, \vec{s} \, [\vec{p}\,\delta(\vec{r}) - \delta(\vec{r})\,\vec{p}\,]$$

$G_F \approx 3 \cdot 10^{-12} \, mc^2 \left(\frac{\hbar}{mc}\right)^3$ Fermi's constant

$\hat{\vec{s}}$ - electron spin

$\hat{\vec{p}}$ - electron momentum

Q_W is the dimentionless weak nuclear charge

$$Q_W = \underset{\#\text{ of } n}{\underset{\uparrow}{-N}} + (1 - 4\sin^2\theta_W)\underset{\#\text{ of } p}{\underset{\uparrow}{Z}}$$

$\sin^2\theta_W \approx 0.23$ - Weinberg mixing angle

$H_W \sim \vec{s} \cdot \vec{p}$ $\qquad \Pi\vec{s} = \vec{s}\,\Pi$

$\qquad\qquad\qquad \Pi\vec{p} = -\vec{p}\,\Pi$

$\hat{\Pi}$ and $\hat{H}_W$ anti-commute in such parity is not conserved interaction

Thus, in principle, the states of different parity are mixed

$$|n\tilde{S}_{1/2}\rangle = |nS_{1/2}\rangle + \frac{\langle n' p_{1/2}|H_W|nS_{1/2}\rangle}{\Delta E} |n' p_{1/2}\rangle$$

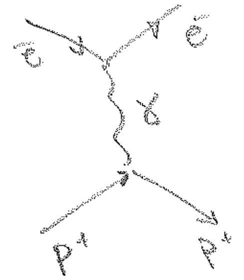
$$= |nS_{1/2}\rangle + i\eta |n' p_{1/2}\rangle$$

$\Delta E = E_s - E_p$

will affect nearly-degenerate states the most!

The value of correction depends on the values of wave-functions at the nuclei ($\delta(\vec r)$)

EM



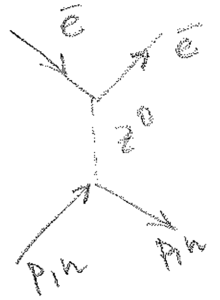
$$\psi_s(\vec r) \approx \frac{\sqrt{Z}}{a_0^{3/2}}\, e^{-Zr/a_0}$$

$$\psi_p(\vec r) \sim \left(\frac{Z}{a_0}\right)^{3/2} \frac{r}{a_0}\, e^{-Zr/a_0} \qquad (=0 \text{ for } \vec r = 0)$$

$$\langle n' P_{1/2} | \delta(\vec r)\, \vec s \vec p \,| n S_{1/2}\rangle = 0$$

$$\langle n' P_{1/2} | \vec s \vec p\, \delta(\vec r) | n S_{1/2}\rangle \sim -i \frac{G_F Q_W \hbar}{mc}\, \psi_s(0) \left.\frac{\partial \psi_p}{\partial r}\right|_0$$

$$= -i \frac{G_F Q_W \hbar}{mc} \frac{\sqrt{Z}}{a_0^{3/2}} \cdot \frac{Z^{3/2}}{a_0^{5/2}} = -i \frac{G_F Q_W \hbar}{mc} \frac{Z^2}{a_0^4}$$



considering $Q_W \sim Z$

$$\langle n' P_{1/2} | H_W | n S_{1/2}\rangle \propto Z^3 \quad \leftarrow \text{need to look at havier atoms (Sm, Dy, Y)}$$

neutral weak interaction

In hydrogen $2S_{1/2}$ & $2P_{1/2}$ states are nearly degenerate (only Lamb shift)
For these state the mixing is

$$|2\tilde S_{1/2}\rangle = |2S_{1/2}\rangle + i\eta\, |2P_{1/2}\rangle$$

$$\eta = -\sqrt{\frac{3}{2}}\, \frac{3\cdot 10^{-12}}{64\pi}\, Q_W\, \alpha^4 mc^2 \frac{1}{\Delta E}$$

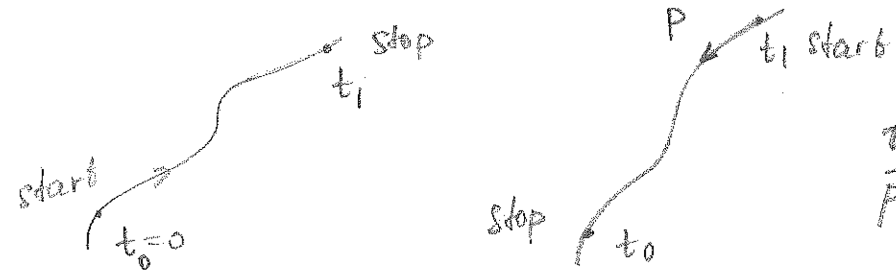
$$Q_W = 1 - 4\sin^2\theta_W \approx 0.08 \qquad \Delta E = 1058\,MHz = 4\cdot 10$$

$$\eta_H \approx -5\cdot 10^{-13}$$

In Cs $\quad Q_W \approx -75 \quad Z = 55, \quad \Delta E = -1.25\,eV$

$$\eta_{Cs} \approx -5\cdot 10^{-12}$$

Time-reversal symmetry
(or reversal of motion)



$t \to -t$
$\vec{p} \to -\vec{p}$

$\vec{r} \to \vec{r}$ even
$\vec{p} \to -\vec{p}$ odd
$\vec{\ell} = \vec{r} \times \vec{p} \to -\vec{r} \times \vec{p}$ odd

$E = \frac{1}{2}mv^2 \to$ even

electric field $\quad \vec{E} = \int \frac{\vec{\rho}(\vec{r}')}{|r-r'|^3}(\vec{r}-\vec{r}')\, dV$ — even

$\vec{B} \propto \int \vec{j}(\vec{r}')$ — odd
$= de\,\vec{v}(\vec{r}')$ as $\vec{j}$ is odd

Lorentz force
$\vec{F} = q\vec{E} + q\vec{v} \times \vec{B}$ — even

We expect that QM will respect
time-reversal symmetry

$$i\hbar \frac{\partial \psi(\vec{r},t)}{\partial t} = \hat{H}\,\psi(\vec{r},t) \qquad \hat{H} \neq \hat{H}(t)$$

$\psi(\vec{r},-t)$ is <u>not</u> a solution, but
$\psi^*(\vec{r},-t)$ still is

$$-i\hbar \frac{\partial \psi^*(\vec{r},t)}{\partial t} = \hat{H}\,\psi^*(\vec{r},t)$$

For the eigenstates

$$\Psi_n(\vec{r},t) = e^{-iE_nt/\hbar}\,\psi_n(\vec{r})$$

$$i\hbar\frac{\partial \Psi_n}{\partial t} = (i\hbar)\left(-\frac{iE_n}{\hbar}\right)e^{-iE_nt/\hbar}\,\psi_n(r) = \hat{H}\psi_n\cdot e^{-iE_nt/\hbar}$$

$$E_n\psi_n(\vec{r}) = \hat{H}\psi_n(\vec{r}) \quad ✓$$

Apply time reversal: $\tilde{\Psi}(r,t) = \Psi(\vec{r},-t) = \psi(\vec{r})\,e^{iE_nt/\hbar}$

$$i\hbar\frac{\partial \tilde{\Psi}_n}{\partial t} = (i\hbar)\left(\frac{iE_n}{\hbar}\right)e^{iE_nt/\hbar}\,\psi_n(\vec{r}) = -E_n\,e^{iE_nt/\hbar}\,\psi_n(\vec{r})$$

$$\neq E_n\psi_n(\vec{r})\,e^{iE_nt/\hbar}$$

but $\hat{H}\tilde{\Psi}^*_n = \hat{H}^\dagger\Psi^*_n = (\hat{H}\Psi_n)^* = E_n\Psi^*_n$

so $\psi^*(r)\,e^{iE_nt/\hbar}$ is a solution of the Schrodinger eqn

Time reversal operator $\hat{\Theta}$

Original state $|\alpha\rangle$

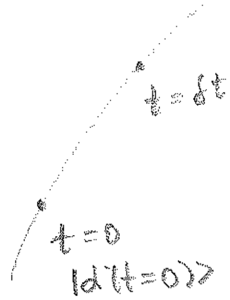
Time reversed state $\hat{\Theta}|\alpha\rangle$

$\hat{\Theta}[\hat{p}|\alpha\rangle] = -\hat{p}\,[\hat{\Theta}|\alpha\rangle]$ or $\hat{\Theta}\hat{p}\hat{\Theta}^{-1} = -\hat{p}$

Similarly $\hat{\Theta}\hat{L}\hat{\Theta}^{-1} = -\hat{L}$ angular momentum

If a hamiltonian preserves time-inversion symmetry, we expect that $\hat{\Theta}\hat{H}\hat{\Theta}^{-1} = \hat{H}$

$[\hat{\Theta}\hat{H}] = 0$ commute

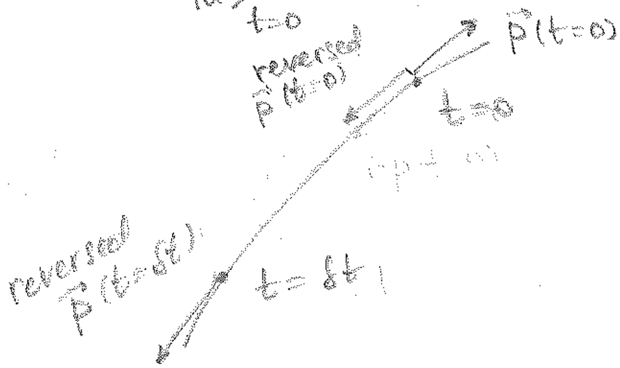


$$|\alpha(t)\rangle = e^{-\frac{i\hat{H}\delta t}{\hbar}}\,|\alpha(t=0)\rangle \approx \left(1 - \frac{i\hat{H}\delta t}{\hbar}\right)|\alpha(t=0)\rangle$$

If the time reversal symmetry stands, then evolving a state and then reversing time should be equivalent to back-tracking the time-reversed state.

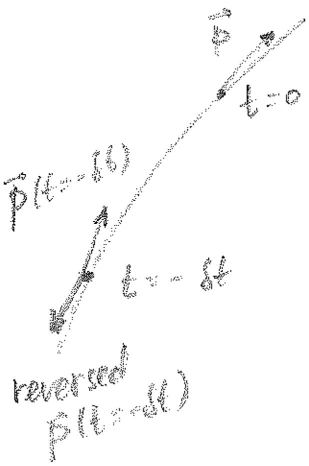
Time reverse & Propagate

$$|\alpha\rangle_{t=0} \longrightarrow \Theta\,|\alpha(t=0)\rangle \longrightarrow \underline{\left(1 - \frac{i\hat{H}\delta t}{\hbar}\right)\hat{\Theta}\,|\alpha(t=0)\rangle}$$



must be equivalent to earlier moment $t = -\delta t$ with $\vec{p}$ reversed

$$\Theta\,|\alpha(t=-\delta t)\rangle = \underline{\hat{\Theta}\left(1 - \frac{i\hat{H}(-\delta t)}{\hbar}\right)|\alpha(t=0)\rangle}$$



$$\hat{\Theta}\left(1+\frac{i\hat{H}\delta t}{\hbar}\right)|\alpha\rangle = \left(1-\frac{i\hat{H}\delta t}{\hbar}\right)\hat{\Theta}|\alpha\rangle$$

$$\hat{\Theta}\cdot i\hat{H}\cancel{\frac{\delta t}{\hbar}}|\alpha\rangle = -i\hat{H}\hat{\Theta}\cancel{\frac{\delta t}{\hbar}}|\alpha\rangle$$

Normally, we would next write $\hat{\Theta}\hat{H} = -\hat{H}\hat{\Theta}$ – $\hat{H}$ & $\hat{\Theta}$ anti-commute but this contradicts our assumption of time reversal symmetry.

$\hat{\Theta}$ is not a linear operator; it is anti-linear and anti-unitary operator

$$\hat{\Theta}(c_1|\alpha\rangle + c_2|\beta\rangle) = c_1^*\hat{\Theta}|\alpha\rangle + c_2^*\hat{\Theta}|\beta\rangle$$

and if

$|\tilde{\alpha}\rangle = \hat{\Theta}|\alpha\rangle$ and $|\tilde{\beta}\rangle = \hat{\Theta}|\beta\rangle$

$\langle\tilde{\beta}|\tilde{\alpha}\rangle = (\langle\beta|\alpha\rangle)^*$

Position space wave-function

$$|\alpha\rangle = \int dV\, \langle\vec{r}|\alpha\rangle\,|\vec{r}\rangle$$

$$\hat{\Theta}|\alpha\rangle = \int dV\, \langle\vec{r}|\alpha\rangle^* \underbrace{\hat{\Theta}|\vec{r}\rangle}_{=|\vec{r}\rangle} = \int dV\langle\vec{r}|\alpha\rangle^*|\vec{r}\rangle$$

$$\psi_\alpha(\vec{r})_{t=0} \xrightarrow{\hat{\Theta}} \psi^*(\vec{r})_{t=0}$$

If $|\alpha_n\rangle$ is non-degenerate

$$H\Theta|\alpha_n\rangle = \Theta H|\alpha_n\rangle = \Theta E_n|\alpha_n\rangle = E_n\Theta|\alpha_n\rangle$$

$\Theta|\alpha_n\rangle$ is also an eigenstate

but then $|\alpha_n\rangle = \Theta|\alpha_n\rangle$

and $\psi_\alpha(\vec{r}) = \psi_\alpha^*(\vec{r}) \Rightarrow \psi_\alpha(\vec{r})$ is real

Similarly

$$|\alpha\rangle = \int d^3\vec{p}\ \langle p|\alpha\rangle\ |\vec{p}\rangle$$

$$\Theta|\alpha\rangle = \int d^3\vec{p}\ \langle p|\alpha\rangle^*\ \underbrace{\Theta|\vec{p}\rangle}_{|-p\rangle} = \int d^3\vec{p}\ \langle -p|\psi\rangle^*\ |\vec{p}\rangle$$

$$\psi_\alpha(\vec{p})_{t=0} \xrightarrow{\Theta} \psi^*(-\vec{p})_{t=0}$$

Angular momentum

$$\Theta\ |l,m\rangle = (|l,m\rangle)^* = (-1)^m\ |l,-m\rangle$$