Physics 622

Problem set 3 *(due February 19)*

Sakurai and Napolitano problems (each problem is 10 points):

5.12, 5.13, 5.18

A1. **Stark effect calculations**. Calculate the shift in energy of the $2P_{3/2}$ state of hydrogen in a very weak static electric field *E*, to second order in *E*, assuming that *E* is small enough so that this shift is much less than the fine-structure splitting between the $2P_{1/2}$ and the $2P_{3/2}$ states. In such second-order perturbation theory here, you can consider only the intermediate state for which the energy-denominator is smallest.

A2. **Effect of a finite proton radius**. Replace the point-like nucleus of a hydrogen atom with a uniform electric charge distribution of radius $R \ll a_0$, where a_0 is Bohr radius. What is the resulting electrostatic potential $V_R(r)$? The difference $\Delta V(r) = V_R(r) - (-e^2/r)$ will be proportional to the assumed extension *R* of the nucleus.

a) Considering $\Delta V(r)$ as a perturbation, calculate the correction to the ground-state energy to the first non-vanishing order in R/a_0. *Hint*: the following integral will most likely will be useful:

$$I_n(\alpha) = \int_0^\alpha x^n e^{-x} dx = n!\left(1 - e^{-\alpha}\sum_{k=0}^{n}\frac{\alpha^k}{k!}\right)$$

b) Do the same for the 2s and 2p states
c) Estimate the relative sizes of the corrections, using $R \sim 10^{-15}$m.

Q1. A repulsive short-range potential with a strongly attractive core can be approximated by a square barrier with a delta function at its center, namely:

$$V(x) = -\frac{\hbar^2 g^2}{2m}\delta(x) + V_0\Theta(a - |x|).$$

Show that there is a negative-energy eigenstate (the ground-state). If E_0 is the ground-state energy of the delta function potential in the absence of the positive potential barrier, show that the ground-state energy of the present system obeys $E \leq E_0 + V_0$