Taylor series:

$$f(x) = f(a) + \sum_{n=1}^{\infty} \frac{f^{(n)}(a)}{n!}(x-a)^n$$

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}$$

$$\cos x = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n}}{(2n)!}$$

$$\sin x = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{(2n+1)!}$$

$$\ln(1+x) = \sum_{n=1}^{\infty} (-1)^{n+1} \frac{x^n}{n}$$

$$(1+x)^p = \sum_{n=0}^{\infty} \binom{n}{p} x^n,$$

$$\binom{n}{p} = \frac{p(p-1)\ldots(p-n+1)}{n!}$$

Fourier series

$$a_n = \frac{1}{l}\int_{-l}^{l} f(x)\cos\frac{n\pi x}{l}\,dx$$

$$b_n = \frac{1}{l}\int_{-l}^{l} f(x)\sin\frac{n\pi x}{l}\,dx$$

$$c_n = \frac{1}{2l}\int_{-l}^{l} f(x) e^{-\frac{in\pi x}{l}}\,dx$$

$$f(x) = \tfrac{1}{2}a_0 + \sum_{n=1}^{\infty} a_n \cos\frac{n\pi x}{l} + \sum_{n=1}^{\infty} b_n \sin\frac{n\pi x}{l}$$

$$f(x) = \sum_{n=-\infty}^{\infty} c_n e^{\frac{in\pi x}{l}}$$

$$\frac{1}{\pi}\int_{-\pi}^{\pi} \cos mx \cos nx\,dx = \delta_{mn}$$

$$\frac{1}{\pi}\int_{-\pi}^{\pi} \sin mx \sin nx\,dx = \delta_{mn}$$

$$\frac{1}{\pi}\int_{-\pi}^{\pi} \cos mx \sin nx\,dx = 0$$

Complex numbers

$$e^{ix} = \cos x + i\sin x$$

$$|z| = \sqrt{x^2+y^2};\ \tan\theta = y/x$$

$$\ln z = Ln|z| + i\theta + i2\pi n$$

Gamma and Beta functions

$$\Gamma(p) = \int_0^{\infty} t^{p-1} e^{-t}\,dt$$

$\Gamma(p+1) = p!$ for integer $p > 0$,

$\Gamma(p+1) = p\Gamma(p)$ for all real p.

$$erf(x) = \frac{2}{\sqrt{\pi}}\int_0^x e^{-t^2}\,dt$$

$$\Gamma(p)\Gamma(1-p) = \frac{\pi}{\sin \pi p}.$$

$$\mathrm{B}(p,q) = \frac{\Gamma(p)\Gamma(q)}{\Gamma(p+q)}.$$

$$n! = \Gamma(n+1) = n^n e^{-n}\sqrt{2\pi n}$$

for $p > 0,\ q > 0$:

$$\mathrm{B}(p,q) = 2\int_0^{\pi/2} (\sin\theta)^{2p-1} (\cos\theta)^{2q-1}\,d\theta,$$

$$\mathrm{B}(p,q) = \int_0^1 x^{p-1}(1-x)^{q-1}\,dx,$$

$$\mathrm{B}(p,q) = \int_0^{\infty} \frac{y^{p-1}}{(1+y)^{p+q}}\,dy$$

Legendre polynomials

Differential equation:

$$(1-x^2)y'' - 2xy' + l(l+1)y = 0, \qquad l = 0,1,2\ldots$$

Normalization and orthogonality

$$\int_{-1}^{1} P_l P_{l'}\,dx = \frac{2}{2l+1}\delta_{ll'}.$$

$$P_l(1) = 1$$

$$P_l(x) = (-1)^l P_l(-x)$$

Rodriges formula

$$P_l(x) = \frac{1}{2^l\, l!}\frac{d^l}{dx^l}\left(x^2-1\right)^l$$

Generation function

$$\Phi(x,h) = \left(1-2xh+h^2\right)^{-1/2} = \sum_{l=0}^{\infty} P_l h^l.$$

Recurrence relations

$$lP_l = (2l-1)xP_{l-1} - (l-1)P_{l-2}$$

$$xP_l' - P_{l-1}' = lP_l,$$

$$P_l' - xP_{l-1}' = lP_{l-1},$$

$$\left(1-x^2\right)P_l' = lP_{l-1} - lxP_l,$$

$$(2l+1)P_l = P_{l+1}' - P_{l-1}'.$$

Associated Legendre polynomials

Differential equation

$$(1-x^2)y''-2xy'+\left\{l(l+1)-\frac{m^2}{1-x^2}\right\}y=0, \qquad l=0,1,2\ldots \quad m=0,\pm1,\ldots\pm l$$

$$P_l(x)=P_{l0}(x),$$

$$P_{lm}(-x)=(-1)^{l+m}P_{lm}(x).$$

Rodriges formula

$$P_{lm}(x)=\left(1-x^2\right)^{m/2}\frac{1}{2^l l!}\frac{d^{l+m}}{dx^{l+m}}\left(x^2-1\right)^l$$

$$P_{l,-m}(x)=(-1)^m\frac{(l-m)!}{(l+m)!}P_{lm}(x)$$

$$\int_{-1}^{1}dxP_{lm}(x)P_{l'm}(x)=\frac{2}{2l+1}\frac{(l+m)!}{(l-m)!}\delta_{ll'}.$$

Spherical harmonics

$$Y_{lm}(\theta,\phi)=(-1)^m\sqrt{\frac{2l+1}{4\pi}\frac{(l-m)!}{(l+m)!}}P_{lm}(\cos\theta)e^{im\phi},$$

Bessel functions

Differential equation

$$x^2y''+xy'+(p^2-x^2)y=0$$

Bessel functions of the first kind

$$J_{\pm p}(x)=\sum_{n=0}^{\infty}\frac{(-1)^n}{\Gamma(n+1)\Gamma(n\pm p+1)}\left(\frac{x}{2}\right)^{2n\pm p}$$

$$J_{-m}(x)=(-1)^m J_m(x)$$

Bessel functions of the second kind

$$N_p(x)=Y_p(x)=\frac{\cos(\pi p)J_p(x)-J_{-p}(x)}{\sin(\pi p)}$$

Generation function

$$e^{\frac{x}{2}(h-\frac{1}{h})}=\sum_{n=-\infty}^{\infty}J_n(x)h^n$$

Recurrence relations

$$J_{p-1}(x)+J_{p+1}(x)=\frac{2p}{x}J_p(x)$$

$$J_{p-1}(x)-J_{p+1}(x)=2J'_p(x)$$

$$\frac{d}{dx}\left[x^pJ_p(x)\right]=x^pJ_{p-1}(x)$$

$$\frac{d}{dx}\left[x^{-p}J_p(x)\right]=-x^{-p}J_{p+1}(x)$$

Hermite polynomials

Differential equation

$$y''-2xy'+2ny=0, \qquad n=0,1,2\ldots$$

Rodriges formula

$$H_n(x)=(-1)^n e^{x^2}\frac{d^n}{dx^n}e^{-x^2}$$

Normalization and orthogonality

$$\int_{-\infty}^{\infty}e^{-x^2}H_n(x)H_m(x)dx=\sqrt{\pi}\,2^n n!\delta_{nm}$$

Generation function

$$\Phi(x,h)=e^{2xh-h^2}=\sum_{n=0}^{\infty}H_n(x)\frac{h^n}{n!}$$

Recurrence relations

$$H_n'(x)=2nH_{n-1}(x)$$

$$H_{n+1}(x)=2xH_n(x)-2nH_{n-1}(x)$$

Laguerre polynomials

Differential equation

$$xy''+(1-x)y'+ny=0, \qquad n=0,1,2\ldots$$

Rodriges formula

$$L_n(x)=\frac{1}{n!}e^{x^2}\frac{d^n}{dx^n}(x^ne^{-x})$$

Normalization and orthogonality

$$\int_{-\infty}^{\infty}e^{-x}L_n(x)L_m(x)dx=\delta_{nm}$$

Generation function

$$\Phi(x,h)=\frac{e^{-xh/1-h}}{1-h}=\sum_{n=0}^{\infty}L_n(x)h^n$$

Recurrence relations

$$L_{n+1}'(x)-L_n'(x)+L_n(x)=0$$

$$(n+1)L_{n+1}(x)-(2n+1-x)L_n(x)+nL_{n-1}(x)=0$$

$$xL_n'(x)-nL_n(x)+nL_{n-1}(x)=0$$