Light - atom interactions

classical e-m field + classical atom	=>	classical e-m field + quantum atom	=>	Quantum EM + quantum atom
can explain absorption, dispersion nonlinear optical phenomena		precision spectroscopy		quantum optics

Electromagnetic wave

Description : Maxwell's equations

Light = mutually induced oscillations of electric and magnetic field

ME in Vacuum (no charges around)

$$\nabla \cdot \vec{B} = 0 \qquad \nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\nabla \cdot \vec{E} = 0 \qquad \nabla \times \vec{B} = \mu_0 \varepsilon_0 \frac{\partial \vec{E}}{\partial t}$$

μ_0 - permeability of free space

ε_0 - permettivity — " —

due to SI units definition

$$\nabla \times (\nabla \times \vec{E}) = -\frac{\partial}{\partial t}(\nabla \times \vec{B}) = -\mu_0 \varepsilon_0 \frac{\partial^2 \vec{E}}{\partial t^2}$$

$$\nabla \times (\nabla \times \vec{E}) = \nabla \underbrace{(\nabla \cdot \vec{E})}_{=0} - \nabla^2 \vec{E}$$

Wave equation $\boxed{\nabla^2 \vec{E} - \mu_0 \varepsilon_0 \frac{\partial^2 \vec{E}}{\partial t^2} = 0}$

Same for $\vec{B}$ (or vector potential $\vec{A}$)

Speed of light in vacuum $c^2 = \frac{1}{\mu_0 \varepsilon_0}$

Linear equation : superposition of solutions is also a solution

Solution form: $\vec{E}(\vec{r},t) = E_0 \underbrace{f(\vec{k}\vec{r} - \omega t)}_{\text{arbitrary function}}$

ω - cyclic frequency [rad/s]
$|\vec{k}| = \omega/c$, $\vec{k}$ - wave vector in the direction of propagation.

Superposition principle: $\vec{E}(\vec{r},t) = \sum_n E_0^{(n)} f(\vec{k}_n \vec{r} - \omega t)$

In many casess
~~Traditionally~~, we will be considering plane waves (flat wavefront, known propagation direction)
For simplicity assuming linear polarization
$\vec{E}_0 = E_0 \vec{e}_x$

Truely plane wave
$\vec{E}(\vec{r},t) = \vec{E}_0 \cos(\vec{k}\vec{r} - \omega t + \varphi)$ $\quad \vec{E}_0 \neq E_0(\vec{r},t)$

Traditionally in optics, a positive/negative frequency decomposition is used

$$\vec{E}(\vec{r},t) = \vec{E}^+(\vec{r},t) + \vec{E}^-(\vec{r},t) = \underbrace{\tfrac{1}{2}\vec{E}_0 e^{i(\vec{k}\vec{r} - \omega t + \varphi)}}_{\vec{E}^+} + \underbrace{\tfrac{1}{2}\vec{E}_0 e^{-i(\vec{k}\vec{r} - \omega t + \varphi)}}_{\vec{E}^-}$$

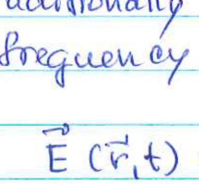
Since $\vec{E}^- = \vec{E}^{+*}$ both the solutions of ME, we can mostly work with one only $(\vec{E}^+)$

$\vec{E} = \vec{E}_0 e^{i\vec{k}\vec{r} - i\omega t}$ $\quad \nabla \cdot \vec{E}_0 = 0 \Rightarrow \vec{E}_0 \cdot \vec{k} = 0 \quad \vec{E}_0 \perp \vec{k}$

$\nabla \times \vec{E} = -\dfrac{\partial \vec{B}}{\partial t}$ $\quad \vec{k} \times \vec{E}_0 = \omega \vec{B}_0 \quad \Rightarrow |\vec{B}_0| = \frac{1}{c}|\vec{E}_0|$

$\vec{B}_0 \perp \vec{E}_0 \perp \vec{k}$

Plane wave is transverse

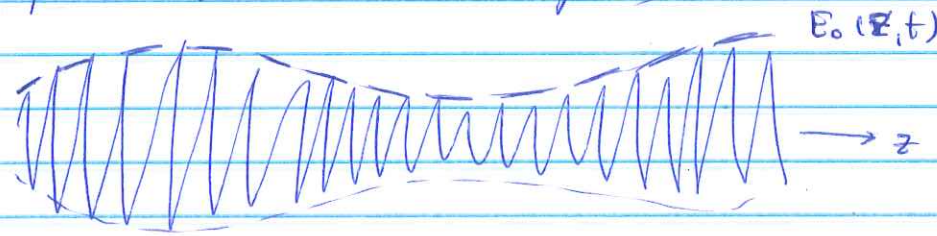


EM wave is never <u>truly</u> plane wave

$$\vec{E}(\vec{r},t) = \underbrace{\tfrac{1}{2}\vec{E}_0(z,t)}_{\text{change in amplitude}}\, e^{ikz - i\omega t + \underbrace{i\varphi(z,t)}_{\text{change in phase}}} \quad (\vec{k} \parallel z)$$

In many cases we can make a <u>slowly varying amplitude and phase approximation</u> (SVAP):

any changes in the amplitude and phase happen <u>much</u> slower than optical period (in time or space)



$$\frac{\partial E}{\partial t} = \frac{\partial}{\partial t}\left(\frac{1}{2}\vec{E}_0(z,t)\, e^{ikz - i\omega t + i\varphi(z,t)}\right) =$$

$$= \left[\underbrace{(-i\omega)\,\vec{E}(z,t)}_{\text{leading term}} + \frac{1}{2}\frac{\partial E_0}{\partial t} e^{ikz - i\omega t + i\varphi} + E(z,t)\, i \frac{\partial \varphi}{\partial t}\right]$$

$$|-i\omega E_0| \gg \left|\partial E_0/\partial t\right| \Rightarrow \left|\frac{\partial E_0}{\partial t}\right| \ll \omega E_0$$

similarly

$$\left|\frac{\partial E_0}{\partial z}\right| \ll k E_0$$

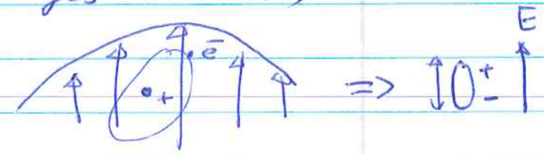
and $\left|\frac{\partial \varphi}{\partial t}\right| \ll \omega$ and $\left|\frac{\partial \varphi}{\partial z}\right| \ll k$

Since $\left|\frac{\partial E_0}{\partial t}\right| \ll \omega E_0$, $\left|\frac{\partial E_0}{\partial z}\right| \ll k E_0$, we can simplify ME

$$\begin{cases} \dfrac{\partial E_0}{\partial z} + \dfrac{1}{c}\dfrac{\partial E_0}{\partial t} = 0 \\ \dfrac{\partial \varphi}{\partial z} + \dfrac{1}{c}\dfrac{\partial \varphi}{\partial t} = 0 \end{cases}$$

SVAP – approximation for ME in vacuum

Electromagnetic field in a medium
(i.e. we have some charges around)

Atoms / molecules = dipoles 

We consider neutral dialectric materials
(i.e. no free charges)
Then electric field can excite oscillations in charge density, creating an oscillating dipole moment $\vec{d}$, at the frequency of the external field

Polarization of the material $\vec{P} = \underbrace{\sum \vec{d}_i}_{\text{inside the unit volume}} = N \underbrace{\langle \vec{d}(t) \rangle}_{\text{average}}$

Maxwell's eqns in a dialectric material

$$\nabla \cdot \vec{B} = 0 \qquad \nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \qquad \vec{D} = \varepsilon_0 \vec{E} + \vec{P} = \varepsilon \varepsilon_0 \vec{E}$$

$$\nabla \cdot \vec{D} = 0 \qquad \nabla \times \vec{H} = \partial \vec{D} / \partial t \qquad \vec{H} = \frac{1}{\mu_0} \vec{B} - \vec{M}$$

For most of the situations we are going to consider, $\vec{M} = 0$ (no magnetic response)

Wave equation in a medium

$$\nabla \times [\nabla \times \vec{E}] = -\frac{\partial}{\partial t} [\nabla \times \vec{B}] = -\mu_0 \frac{\partial}{\partial t} [\nabla \times \vec{H}] = -\mu_0 \frac{\partial^2 \vec{D}}{\partial t^2}$$

$$\nabla \times [\nabla \times \vec{E}] = \nabla(\nabla \cdot \vec{E}) - \nabla^2 \vec{E} \approx -\nabla^2 \vec{E}$$

Strictly speaking $\nabla \cdot \vec{E} \neq 0$ ($\nabla \vec{D} = 0$), but for a transverse EM waves $\vec{k} \cdot \vec{E} \to 0$
(identically zero for a plane wave)

Wave eqn in a medium

$$-\nabla^2 \vec{E} + \frac{1}{c^2} \frac{\partial^2 \vec{E}}{\partial t^2} = -\mu_0 \frac{\partial^2 \vec{P}}{\partial t^2}$$