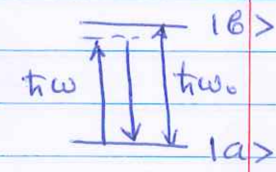
Jaynes - Cummings model



$$\hat{H}_{int} = \hbar g \left(\hat{\sigma}_+ \hat{a} + \hat{\sigma}_- \hat{a}^+ \right)$$

$$\hat{H}_{total} = \frac{1}{2}\hbar\omega_0 \hat{\sigma}_3 + \hbar\omega \hat{a}^+\hat{a} + \hbar g \left(\hat{\sigma}_+ \hat{a} + \hat{\sigma}_- \hat{a}^+ \right)$$

In general if at $t=0$ the atom is in the excited state $|b\rangle$ and the state of light is $|\psi_{ph}\rangle = \sum_{n=0}^{\infty} c_n |n\rangle$

Then each Foch state $|n\rangle$ evolves by going through quantum Rabi flopping

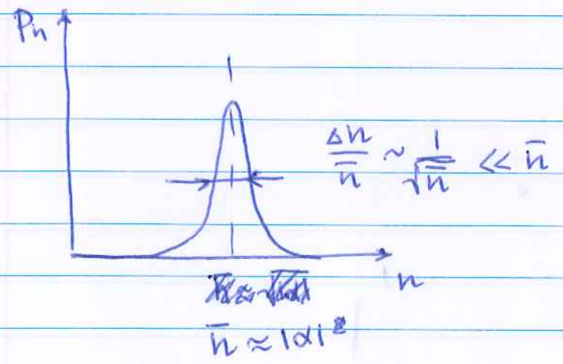
$$|n\rangle \longrightarrow \cos(gt\sqrt{n+1})|b\rangle - i\sin(gt\sqrt{n+1})|a\rangle$$

Thus a general state evolves

$$|\psi(t)\rangle = \sum_{n=0}^{\infty} \left\{ c_n \cos(gt\sqrt{n+1})|n\rangle|b\rangle - i\, c_{n-1} \sin(gt\sqrt{n}) |n\rangle|a\rangle \right\}$$

If the initial state is a coherent state $|\alpha\rangle$, then the system evolves as follows

$$|\psi(t)\rangle = \sum_n e^{-|\alpha|^2/2} \frac{\alpha^n}{\sqrt{n!}} \left(i\sin(g\sqrt{n+1}\,t)|n+1\rangle|a\rangle + \cos(g\sqrt{n+1}\,t)|n\rangle|b\rangle \right)$$



So if we neglect the amplitude fluctuations and replace the realistic photon number distribution with $P_n = \delta(n-\bar{n})$, i.e $\sqrt{n+1} \approx \sqrt{\bar{n}}$

$$|\psi(t)\rangle \approx \sum_n e^{-|\alpha|^2/2} \frac{\alpha^n}{\sqrt{n!}} \left(i\sin(g\sqrt{\bar{n}}t)\,|n\rangle|a\rangle + \cos(g\sqrt{\bar{n}}t)\,|n\rangle|b\rangle \right)$$

$$\approx \left[\left(i\sin(g\sqrt{\bar{n}}t)\,|a\rangle + \cos(g\sqrt{\bar{n}}t)\,|b\rangle \right] |\alpha\rangle$$

identical to classical Rabi flopping

This approximation works until $g\tau\left(\sqrt{\bar{n}+\Delta n} - \sqrt{\bar{n}-\Delta n}\right)$

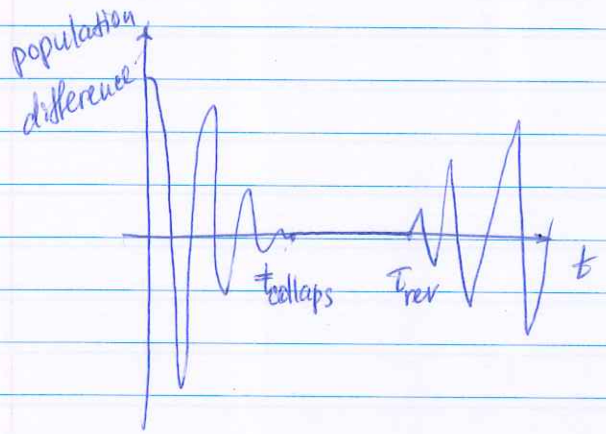
$$\approx g\tau\sqrt{\bar{n}}\left(\left(1+\frac{\Delta n}{2\bar{n}}\right) - \left(1-\frac{\Delta n}{2\bar{n}}\right) \right) \approx g\tau \left(\frac{\Delta n}{\sqrt{\bar{n}}}\right) \sim 1$$

$\tau_{collapse}$ => phase spread ~ 1 $\quad \tau_{collaps} \approx 1/g$

Revival → all sine waves come in phase again

$$\left(g\sqrt{\bar{n}+1} - g\sqrt{\bar{n}}\right)\tau_{rev} = 2\pi \quad \text{(or } 4\pi, 6\pi \text{ for later revivals)}$$

$$g\sqrt{\bar{n}}\left(1+\frac{1}{2\bar{n}} - 1\right)\tau_{rev} = 2\pi \quad \Rightarrow \quad \tau_{rev} \approx \frac{2\pi\sqrt{\bar{n}}}{g} = \frac{2\pi|\alpha|}{g}$$



In fact we can write

$$gt\sqrt{n+1} \approx gt\sqrt{\bar{n}}\left(\frac{\bar{n}+n}{2\bar{n}}\right)$$

Then $$igt\sqrt{n+1} = \frac{1}{2}\left(e^{i\varphi_n} - e^{-i\varphi_n}\right) \approx$$

$$\approx e^{ig\sqrt{\bar{n}}\,t/2}\, e^{ignt/2\sqrt{\bar{n}}} - e^{-ig\sqrt{\bar{n}}\,t/2}\, e^{-ignt/2\sqrt{\bar{n}}}$$

and

$$\sum_n e^{-|\alpha|^2/2} \frac{\alpha^n}{\sqrt{n!}}\, i \sin g\sqrt{n+1}\, t\, |a\rangle|n+1\rangle \approx$$

$$\approx \frac{1}{2}|a\rangle \sum_n e^{-|\alpha|^2/2}|n+1\rangle\left(e^{ig\sqrt{\bar{n}}\,t} \frac{(\alpha e^{igt/2\sqrt{\bar{n}}})^n}{\sqrt{n!}} - e^{-ig\sqrt{\bar{n}}\,t/2} \frac{(\alpha e^{-igt/2\sqrt{\bar{n}}})^n}{\sqrt{n!}}\right.$$

$$= \frac{1}{2}|a\rangle\left(e^{ig\sqrt{\bar{n}}\,t/2}|\alpha e^{igt/2\sqrt{\bar{n}}}\rangle - e^{-ig\sqrt{\bar{n}}\,t/2}|\alpha e^{-igt/2\sqrt{\bar{n}}}\rangle\right)$$

for the cos term → almost the same, but with "+".

$$|\psi(t)\rangle = \frac{1}{\sqrt{2}}\left(|b\rangle|\alpha_+\rangle + |a\rangle|\alpha_-\rangle\right)$$

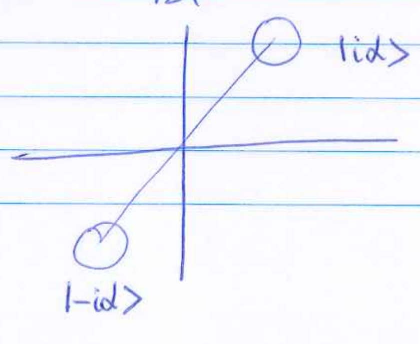
$$|\alpha_\pm\rangle = \frac{1}{\sqrt{2}}\left(e^{ig\sqrt{\bar{n}}\,t/2}|\alpha e^{igt/2\sqrt{\bar{n}}}\rangle \pm e^{-ig\sqrt{\bar{n}}\,t/2}|\alpha e^{-igt/2\sqrt{\bar{n}}}\rangle\right)$$

Similar to Shrodinger cat's states, but evolving in time

for $t \approx t_{rev}/2 = \dfrac{\cancel{2}\pi\sqrt{\bar{n}}}{g}$ $\qquad \dfrac{g\sqrt{\bar{n}}\,T_{rev}}{2} \approx \pi\sqrt{\bar{n}}/2$

$$gt/2\sqrt{\bar{n}} \approx \frac{\pi}{2}$$

$$|\alpha_\pm\rangle \approx \frac{1}{\sqrt{2}}\left(|e^{i\pi/2}\alpha\rangle \mp |\alpha e^{-i\pi/2}\rangle\right) = \frac{1}{\sqrt{2}}\left(|i\alpha\rangle \mp |-i\alpha\rangle\right)$$



exact cat states

At $t \approx T_{rev}/2$ the atom-light system is in an entangled state:

$$|\psi(t)\rangle = \frac{1}{\sqrt{2}}\left(|b\rangle|\alpha_+\rangle + |a\rangle|\alpha_-\rangle\right)$$

To obtain the particular cat state for an optical field, a state of atom can be measured, Finding an atom in state $|b\rangle$ collapses the wavefunction and projects the optical field into state $|\alpha_+\rangle$, and finding an atom in state $|a\rangle$ produces $|\alpha_-\rangle$ optical cat state.