Basics of a laser operation

Necessary components : amplifying medium
feedback

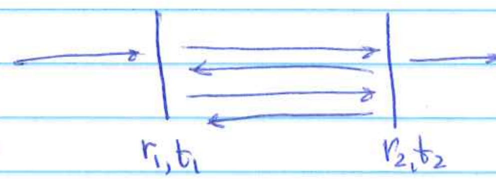
We need population inversion to achieve amplification

Gain coefficient $g = \frac{k}{2} \frac{\wp_{ab}^2}{\varepsilon_0 \hbar \gamma_{ab}} N \frac{\mathcal{L}(\Delta)}{1 + I \cdot \mathcal{L}(\Delta)}$

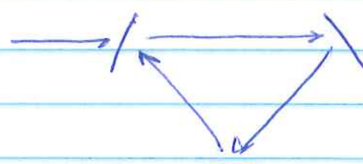
$I(z) = e^{gz} I(0)$ $N = \rho_{aa}^{(0)} - \rho_{bb}^{(0)}$ – population inversion

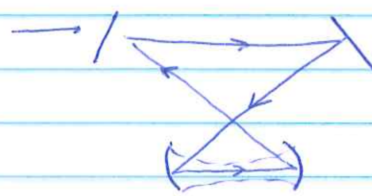
Feedback is provided by an optical cavity

two - mirror configuration
Fabri - Perot resonator



ring cavity
(running-wave resonators)





bow-tie cavity

Important properties of a resonator :
optical field of a certain frequencies can circulate inside
Quality factor of a resonator

$$Q = \frac{\text{stored energy}}{\text{dissipated power}}$$

If L is a fractional loss per round trip, ~~then the fractional loss per~~ and the time of a round-trip is $T = P_{opt}/c$ (where P_{opt} is an optical (perimeter) length of a cavity)

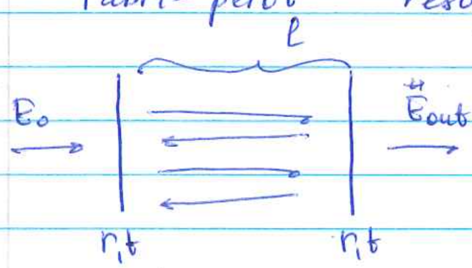
Then $$\frac{d\mathcal{E}}{dt} = -\frac{L\mathcal{E}}{T} = -\frac{cL}{P_{opt}}\mathcal{E}$$ energy stored

Photon lifetime $t_c = \frac{P_{opt}}{cL}$

$$Q = \frac{\omega \cdot \mathcal{E}}{-d\mathcal{E}/dt} = \omega \cdot t_c \qquad \left[\frac{d\mathcal{E}}{dt} = -\frac{\omega}{Q}\mathcal{E}\right]$$

Sources of optical losses
- imperfect reflection of mirrors (accessary eval)
- absorption and scattering in the mirrors and active medium
- diffraction losses

Fabri-perot resonator



no extra losses

$$I_{tot} = I_0 \frac{t^2}{1 - r^2\cos 2kl}$$

$P_{opt} = 2l$ Resonant conditions $2kl = 2\pi \cdot m$
$m = 0, 1, 2 \ldots$

$$\frac{2\pi}{\lambda} \cdot l = \pi m \qquad l_m = m\frac{\lambda}{2}$$

or if the laser frequency is tunable

$$\frac{\omega_m}{c} l = \pi m \qquad \omega_m = \frac{\pi c}{l} \cdot m \quad \text{or} \quad f_m = \frac{c}{2L} m$$

Equidistant modes $\Delta\omega = \frac{c}{2l}$ free spectral range

In case of additional distributed losses α

$$t_c = \frac{2l}{c(2\alpha l + (1 - \sqrt{r_1 r_2}))} = \frac{2l/c}{2\alpha l + (1 - \sqrt{R_1 R_2})}$$

Typically, each mode may have associated transverse structure, and the spectrum of the transverse modes can be not degenerate

Electric field inside a laser

$$E(z,t) = \frac{1}{2} \sum_m E_m(t)\, e^{-i\omega_m t + \varphi_m}\, U_m(z) + c.c$$

Multimode operation, $U_m(z)$ – ~~sp~~ a function describing spatial variation of e-m field

~~for wave~~

Fabri-perot cavity: standing wave

$$U_m(z) = \sin k_m \cdot z$$

Running wave (ring cavity)

$$U_m(z) = e^{ik_m z}$$

In a more rigorous treatment, this function will also include transverse ~~intensity~~ mode distribution

e.g. $U_{mst}(z,x,y) = \sin k_{mst} \cdot z \quad H_{st}(x,y)$ and φ_{mst} can be different for different transverse modes.

Correspondingly, the induced polarization

$$P(z,t) = \frac{1}{2} \sum_m P_m(t)\, e^{-i\omega_m t + \varphi_m}\, U_m(z) + c.c$$

nator.

If the atomic resonance frequency ν_0 does not coincide with the passive resonance frequency ν_m, the laser frequency will, according to (9.1-21), be shifted away from ν_m *toward* ν_0. This phenomenon is called "frequency pulling." Since typically $\Delta\nu_{1/2} \ll \Delta\nu$, the laser tends to oscillate near ν_m.

Example: Frequency Pulling in a He-Ne Laser. In a typical He-Ne 0.6328 μm laser, as an example, we have $l = 30$ cm, $R = 0.99$, $\Delta\nu \simeq 1.5 \times$

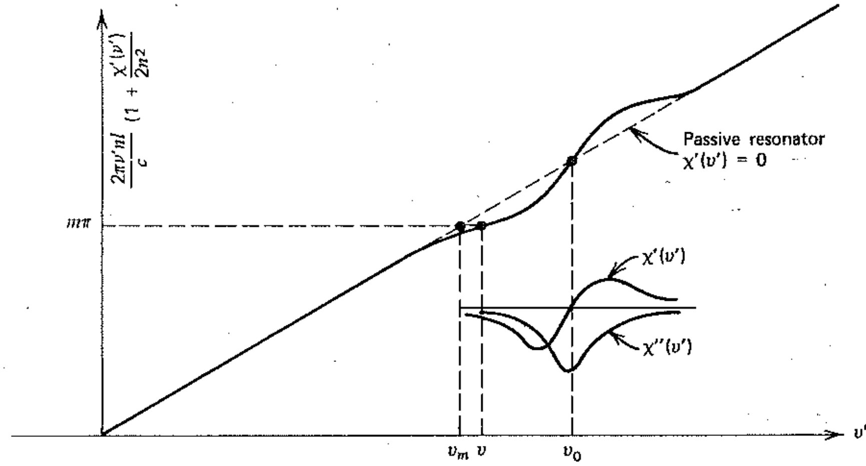


FIGURE 9.2 A graphical illustration of the laser frequency condition [Eq. 9.1-17) showing how the atomic dispersion $\chi'(\nu)$ "pulls" the laser oscillation frequency ν from the passive resonator value ν_m toward that of the atomic resonance at ν_0.

Single-mode operation $(\omega = \omega_m)$

$$\frac{\partial E_m}{\partial t} = -\frac{\omega_m}{2\varepsilon_0} \operatorname{Im} \mathcal{P}_m \underbrace{- \frac{\omega_m}{2Q_m} E_m}_{\text{losses in the cavity}}$$

$$\omega + \frac{\partial \varphi_m}{\partial t} = \omega_m^{(cavity)} - \frac{\omega_m}{2\varepsilon_0} \frac{1}{E_m} \operatorname{Re}(\mathcal{P}_m)$$

$$\omega_m^{(cavity)} = c \cdot k_m$$

or, substituting $\mathcal{P}_m = \varepsilon \chi_m E_m$

$$\frac{\partial}{\partial t} E_m = -\frac{\omega_m}{2Q_m} E_m - \frac{\omega_m}{2} \chi_m'' E_m$$

$$\frac{\partial \varphi_m}{\partial t} + \omega = \omega_m^{(cavity)} - \frac{\omega_m}{2} \chi_m'$$

Or, rewriting for the mode energy

$$\frac{\partial \mathcal{E}_m}{\partial t} = \underbrace{-\frac{\omega_m}{Q_m} \mathcal{E}_m}_{\substack{\text{cavity loss} \\ \text{per unit time}}} \underbrace{- \omega_m \chi_m'' \mathcal{E}_m}_{\substack{\text{medium gain} \\ \text{per unit time}}} \quad (\text{if } \chi_m'' < 0)$$

The equation for the phase shows that the dispersion of the medium may change the resonant conditions for the laser frequency. In a steady-state $\dot{\varphi}_m = 0$

$$\omega_m = \omega_m^{(cavity)} - \frac{\omega_m}{2} \chi_m'$$

$$\chi_m' \propto (+\Delta) = (\omega_m - \omega_{ab})$$

and

$$\omega_m = \omega_m^{(cavity)} + (\omega_{ab} - \omega_m) \left\{ \begin{array}{l} \text{positive} \\ \text{combination} \\ \text{of parameters} \end{array} \right\}$$

The generation frequency is "pulled" toward the frequency of the atomic resonance

Steady-state laser operation

From last lecture in a two-level system

$$\chi = -i \frac{\wp_{ab}^2}{\varepsilon_0 \hbar} N \boxed{\frac{1}{1+I_0 \mathcal{L}(\Delta)}} \frac{1}{\gamma_{ab}+i\Delta} \qquad \Delta = \omega_{ab} - \omega$$

saturation

Gain parameter $g_m = \dfrac{\omega_m \wp_{ab}^2}{2\varepsilon_0 \hbar \gamma_{ab}} N \mathcal{L}(\Delta_m) \qquad \Delta_m = \omega_{ab} - \omega_m$

Saturation: we derived it for the running wave

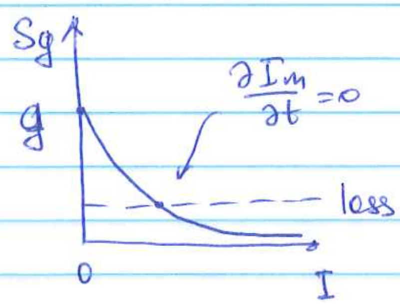
~~Running~~ $$S(I_m) = \frac{1}{1+I_m \mathcal{L}(\Delta_m)}$$

Saturation for the standing wave

$$S(I_m) = \frac{2}{I_m \mathcal{L}(\Delta_m)} \left[1 - \frac{1}{\sqrt{1+I_m \mathcal{L}(\Delta_m)}} \right]$$

And the equation for the normalized mode intensity

$$\frac{\partial}{\partial t} I_m = 2 I_m \left(g_m S(I_m) - \frac{\omega}{2Q_m} \right)$$



Steady-state solution

Saturated gain = loss

$$g_m S(I_m) = \frac{\omega}{2Q_m}$$

This expression provides the way to find the steady-state laser power

~~Considering a simplified case called ($\omega_{ab}=\omega_m$)~~

Threshold behavior: unsaturated gain is equal to the losses (min requirements for laser operation)

$$g_m = \frac{\omega_m}{2Q_m} = \frac{\omega_m \wp_{ab}^2}{2\varepsilon_0 \hbar \gamma_{ab}} N_T \mathcal{L}(\Delta_m)$$

$=1$ since typically we define threshold for resonant case

N_T - minimum required population inversion

$$N_T = \frac{\varepsilon_0 \hbar \gamma_{ab}}{\wp_{ab}^2 Q_m}$$

The higher is Q_m factor, the lower is the required inversion

We can then express the gain in terms of N_T

$$g_m = \frac{\omega}{2Q_m} \frac{N}{N_T} \mathcal{L}(\Delta)$$

When operating at threshold, the ~~laser~~ circulating laser energy is infinitesimally small.

Above - threshold operation

$$g_m S(I_m) = \frac{\omega}{2Q_m} \Rightarrow S(I_m) = \frac{1}{g_m} \frac{\omega}{2Q_m} = \frac{N_T}{N\mathcal{L}(\Delta m)}$$

for a running-wave cavity $\qquad S(I) = \frac{1}{1+I\mathcal{L}}$

$$I_m = \frac{N/N_T \cdot \mathcal{L}(\Delta m) - 1}{\mathcal{L}(\Delta m)}$$

Standing wave saturation

$$I_m = \frac{N}{N_T} - \frac{1}{4\mathcal{L}} - \frac{1}{4\mathcal{L}}\sqrt{10\frac{N}{N_T}\mathcal{L} + 1}$$