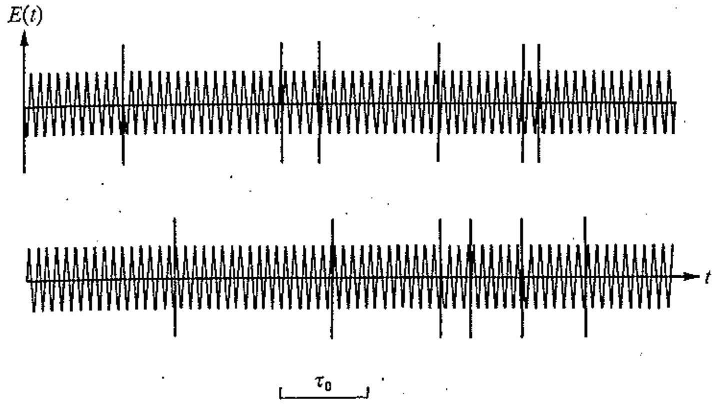
Coherence

A process is coherent if it is characterized by the existance of some well-defined determenistic phase relationship, or, in other words, if some phase is not subject to random noise.

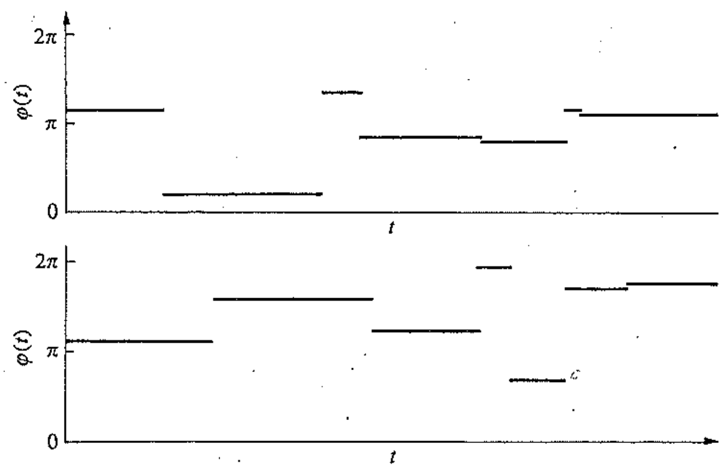
$$E(z,t) = E_0 \, e^{ikz - i\omega t + \varphi(t)}$$

Ideal plane wave $\varphi(t) = \text{const}$

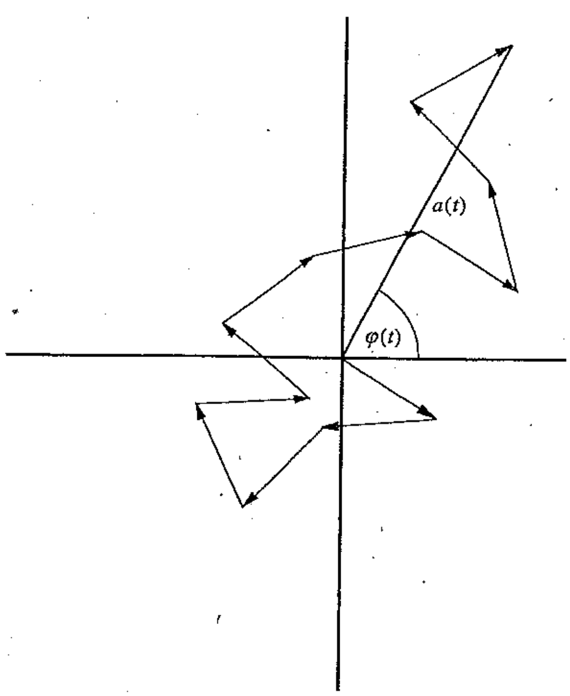
Realistically, $\varphi(t)$ remains constant only for a period of time, and then changes arbitrary



ig. 3.1. The electric-field amplitude of the wave train radiated by a single atom. The vertical lines represent collisions separated by periods of free flight with the mean duration τ_0 indicated. The quantity $\omega_0\tau_0$ is chosen unrealistically small in order to show the random phase changes caused by the collisions.



. Time dependence of the phase angle of the wave train illustrated in Fig.



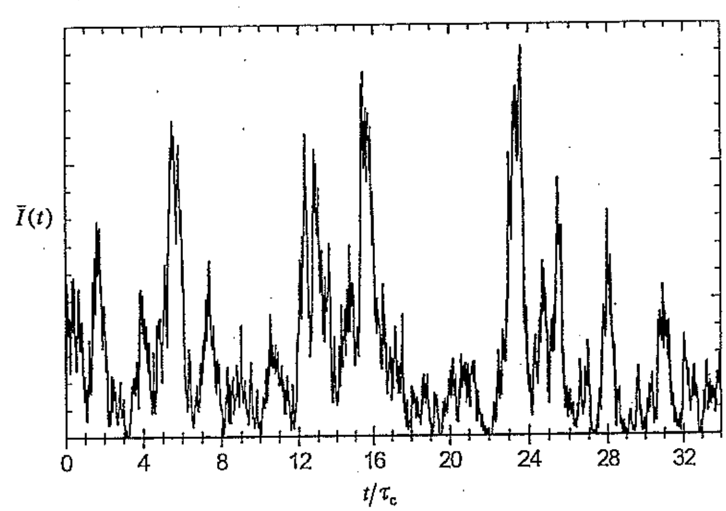
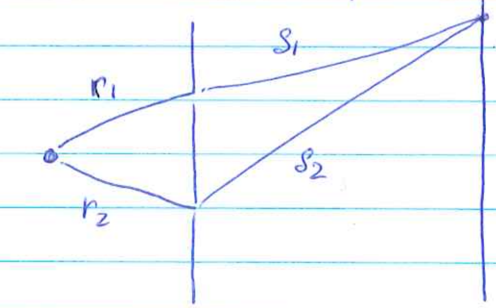


Fig. 3.4. Time series of the cycle-averaged intensity for a collision-broade chaotic light beam obtained from a computer simulation [1]. The time scal normalized by the coherence time τ_c, taken equal to the mean time betw collisions. (After [1])

Double-slit experiment



$$E_{total}(t) = E_1(t-t_1) + E_2(t-t_2)$$

$$t_{1,2} = t - s_{1,2}/c$$

$$\tau = t_2 - t_1 = s_2 - s_1/c$$

delay b/w two paths

Intensity of the total e-m field

$$|E_{total}|^2 = |E_1(t-t_1)|^2 + |E_2(t-t_2)|^2 + 2\,\mathrm{Re}\,(E_1^* E_2)$$

To describe light with noise we need to use statistical approach, repeating the measurements many times and averaging the result

$$\langle |E_{total}|^2 \rangle = \langle |E_1(t-t_1)|^2 \rangle + \langle |E_2(t-t_2)|^2 \rangle + 2\,\mathrm{Re}\,\langle E_1^*(t-t_1) E_2(t-t_2) \rangle$$

$$E(t) = |E(t)|\, e^{ikz - i\omega t + i\varphi(t)}$$

$$|E_{1,2}|^2 = E_{1,2}^* E_{1,2}; \quad E_1^*(t-t_1) E_2(t-t_2) = |E_1(t)|\,|E_2(t)|\, e^{i\varphi(t_2) - i\varphi(t_1)}$$

$$\langle |E_{tot}|^2 \rangle = \langle |E_1|^2 \rangle + \langle |E_2|^2 \rangle + 2 \langle |E_1| \rangle \langle |E_2| \rangle \cos(\varphi(t_2) - \varphi(t_1))$$

First-order correlation function

$$G^{(1)}(t_1, t_2) = \langle E^-(t_1) E^+(t_2) \rangle$$

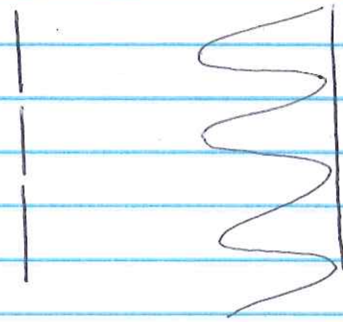
$$\langle |E_{tot}|^2 \rangle = G^{(1)}(t_1,t_1) + G^{(1)}(t_2,t_2) + \underbrace{2\,|G^{(1)}(t_1,t_2)| \cos\varphi(t_1,t_2)}_{\text{term responsible for interference}}$$

Ideal laser light (=coherent light)

$\varphi = \text{const}$, $\varphi(t_1) - \varphi(t_2) = (S_1 - S_2)/c$

$\cos(\varphi_1 - \varphi_2) = \cos(S_1 - S_2/c)$

normal interference



It is convenient to characterize the contrast of fringes using visibility

$$V = \frac{I_{max} - I_{min}}{I_{max} + I_{min}}$$

For an ideal coherent source $I_{min} = 0$

$V = 1$

In general case

$$I_{\substack{max \\ min}} = \langle G^{(1)}(t_1,t_1)\rangle + \langle G^{(1)}(t_2,t_2)\rangle \pm 2|G^{(1)}(t_1,t_2)|$$

$$V = \frac{2|G^{(1)}(t_1,t_2)|}{G^{(1)}(t_1,t_1) + G^{(1)}(t_2,t_2)}$$

In majority of cases for a ~~stationary~~ cw optical source $\langle |E(t-t_1)|^2\rangle = \langle |E(t-t_2)|^2\rangle$

and $G^{(1)} = \langle E_1^*(t-t_1) E_2(t-t_2)|\rangle = \langle E^*(t) E(t-\tau)\rangle$

$\tau = t_2 - t_1$

Normalized first-order temporal coherence function

$$g^{(1)}(\tau) = \frac{\langle E^*(t) E(t+\tau)\rangle}{\langle E^*(t) E(t)\rangle}$$

$$V = |g^{(1)}(\tau)|$$

For ideal coherent light $g^{(1)}(\tau) = 1$ for any τ

In general $0 \leq |g^{(1)}(\tau)| \leq 1$

Random light – collection of independent oscillators with random phase distribution

$$E(t) = E_1(t) + E_2(t) + \dots = E_0 e^{-i\omega_0 t} \{ e^{i\varphi_1(t)} + e^{i\varphi_2(t)} + \dots e^{i\varphi_N(t)} \}$$

$$\langle E^*(t) E(t+\tau) \rangle = |E_0|^2 e^{-i\omega\tau} \Big\langle \left(e^{-i\varphi_1(t)} + e^{-i\varphi_2(t)} + \dots e^{-i\varphi_N(t)} \right) \times$$

$$\times \left(e^{i\varphi_1(t+\tau)} + e^{i\varphi_2(t+\tau)} + \dots e^{i\varphi_N(t+\tau)} \right) \Big\rangle$$

By taking an ensemble average only same-oscillator correlations survive

$$\langle E^*(t) E(t+\tau) \rangle = |E_0|^2 e^{-i\omega\tau} \underbrace{\sum_{i=1}^{N} \langle e^{i\varphi_i(t+\tau) - i\varphi_i(t)} \rangle}_{\text{single-atom correlation}}$$

We will assume that the probability of an oscillator to maintain its phase for time b/w τ and $\tau + d\tau$ is

$$p(\tau) d\tau = \frac{1}{\tau_0} e^{-\tau/\tau_0} dt \qquad \tau_0 \text{ – average coherence time}$$

$$\langle e^{i\varphi_i(t-\tau) - i\varphi_i(t)} \rangle = \begin{cases} 1 & \text{phase didn't jump} \\ 0 & \text{phase jumped} \end{cases}$$

$$= \int_{\tau}^{\infty} p(\tau') d\tau = \quad \text{– probability that the phase is maintained beyond } t = \tau$$

$$= e^{-\tau/\tau_0}$$

$$\langle E^*(t) E(t+\tau) \rangle = N |E_0|^2 e^{-i\omega\tau} \, e^{-\tau/\tau_0}$$

$$g^{(1)}(\tau) = e^{-i\omega\tau - \tau/\tau_0}$$

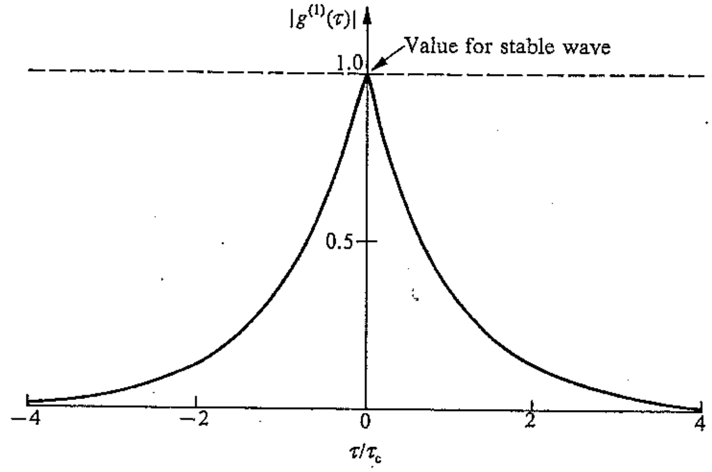


Fig. 3.8. The modulus of the degree of first-order coherence of chaotic light with a Lorentzian frequency spectrum.

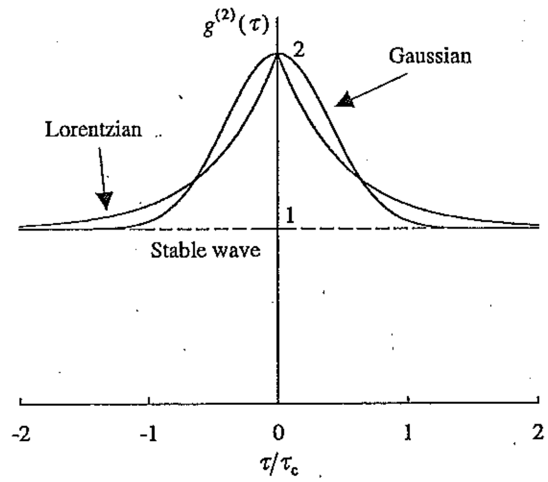
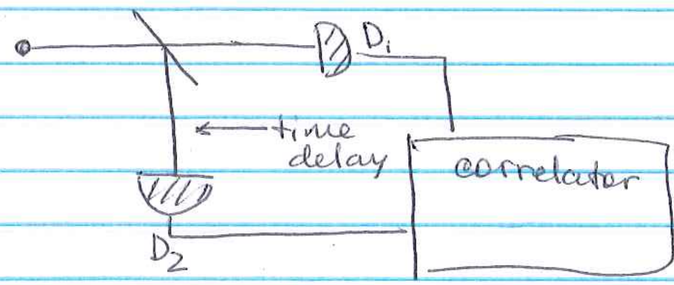


Fig. 3.12. Degrees of second-order coherence of chaotic light having Gaussian and Lorentzian frequency distributions with coherence time τ_c. The dashed line shows the constant unit second-order coherence of a classical stable wave.

Second-order coherence

Hanbury-Brown-Twiss experiment

50:50 beamsplitter



$$G^{(2)}(\tau) = \langle I(t) I(t+\tau)\rangle = \langle E^*(t) E^*(t+\tau) E(t+\tau) E(t)\rangle$$

Normalized second-order correlation function

$$g^{(2)}(\tau) = \frac{G^{(2)}(\tau)}{|G^{(1)}(0)|^2} = \frac{\langle I(t)\, I(t+\tau)\rangle}{I^2}$$

Ideal coherent light $E = E_0 e^{-i\omega t + \varphi} \Rightarrow I(t) = I(t+\tau)$

$$g^{(2)}(\tau) = 1 \quad \text{for any } \tau$$

$$g^{(2)}(0) = \frac{\langle I^2(t)\rangle}{\langle I(t)\rangle^2} = 1 + \frac{\langle I^2\rangle - \langle I\rangle^2}{\langle I\rangle^2} = 1 + \frac{\langle (I - \langle I\rangle)^2\rangle}{\langle I\rangle^2}$$

Since $(I - \langle I\rangle)^2 > 0$ $\quad g^{(2)}(0) \geq 1$ for classical light

and since $\langle I(t)\, I(t+\tau)\rangle \leq \langle I(t)^2\rangle$

$$g^{(2)}(\tau) \leq g^{(2)}(0)$$

Thermal light

$$E(t) = \sum_{i=1}^{N} E_i(t)$$

$$\langle E^*(t) E^*(t+\tau) E(t+\tau) E(t) \rangle = \sum_{i=1}^{N} \langle E_i^*(t) E_i^*(t+\tau) E_i(t+\tau) E_i(t) \rangle +$$

$$+ \sum_{\substack{i,j=1 \\ i \neq j}}^{N} \left\{ \langle E_i^*(t) E_j^*(t+\tau) E_j(t+\tau) E_i(t) \rangle + \langle E_i^*(t) E_j^*(t+\tau) E_i(t+\tau) E_j(t) \rangle \right\}$$

all other term vanish because of the random phases

$$= N \langle E_i^*(t) E_i^*(t+\tau) E_i(t+\tau) E_i(t) \rangle + N(N-1) \left\{ \langle E_i^*(t) E_i(t) \rangle^2 + \right.$$
$$\left. + |\langle E_i^*(t) E_i(t+\tau) \rangle|^2 \right\}$$

$$\langle E_i^*(t) E_i(t) \rangle = G_i^{(1)}(0) = I \qquad N \gg 1$$
$$\langle E_i^*(t) E_i(t+\tau) \rangle = G_i^{(1)}(\tau) \qquad N(N-1) \approx N^2 \gg N$$

$$G^{(2)}(\tau) = N^2 (G_i^{(1)}(0) + G_i^{(1)}(\tau))$$
$$g^{(2)}(\tau) = 1 + |g^{(1)}(\tau)|^2$$

$g^{(2)}(0) = 2$ for the thermal light

$$g^{(2)}(\tau) = 1 + e^{-2|\tau|/\tau_0}$$