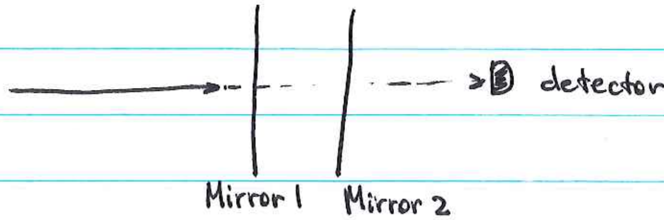
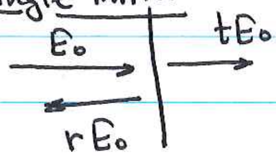
Multi-pass interferometers (cavities)

Fabri-Perot interferometer



Light can bounce between the two mirrors and interfere with itself

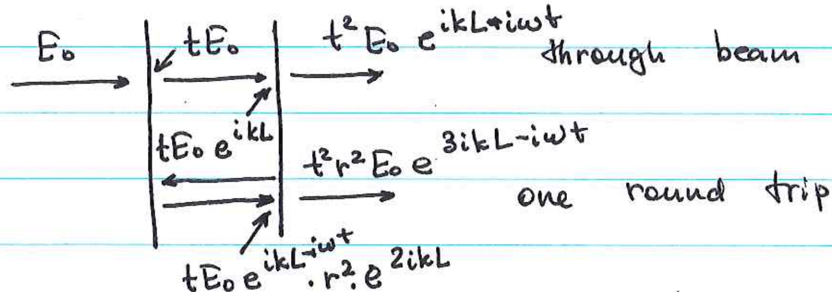
Single mirror



t, r – transmission and reflection coefficients

For a good mirror $r \sim 1$, $t \ll 1$

Two mirrors



$$E_{tot} = \mathrm{Re}\{t^2 E_0 e^{ikL-i\omega t} + t^2 r^2 E_0 e^{3ikL-i\omega t} + t^2 r^4 E_0 e^{5ikL-i\omega t} + \dots\} =$$

$$= \mathrm{Re}\{t^2 E_0 e^{ikL} \underbrace{(1 + r^2 e^{2ikL} + r^4 e^{4ikL} + \dots)}_{\text{interference between multy pass beams}}\}$$

$$= \mathrm{Re}\left\{\frac{t^2 E_0 e^{ikL-i\omega t}}{1 - r^2 e^{2ikL}}\right\} = \frac{t^2 E_0 \cos(kL - \omega t)}{1 - r^2 \cos 2kL}$$

$$I_{tot} = \langle E_{tot}^2 \rangle_{time} = \frac{1}{2} E_0^2 \frac{t^2}{1 - r^2 \cos 2kL} = I_0 \frac{t^2}{1 - r^2 \cos 2kL}$$

<u>Resonance conditions</u> (all multipass beams interfere constructively): $\cos 2kL = 1$ $\quad 2kL = 2\pi m$

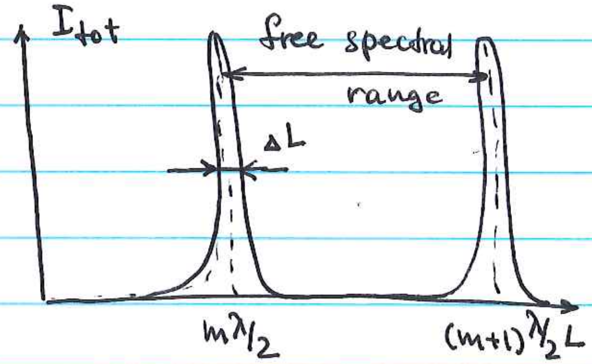
$$\frac{2\pi L}{\lambda} = \pi m$$

$$L = m \cdot \frac{\lambda}{2}$$

Under these conditions each ~~consecu~~ consecutive reflection is shifted in phase by integer # of wavelength.

Then $\quad I_{tot} = I_0 \dfrac{t^2}{1-r^2} = I_0 \quad (\text{if } t^2+r^2=1)$

all light is transmitted through a pair of very reflecting mirrors!



$$\Delta L = \frac{\lambda}{4\pi} \frac{1-r^2}{r}$$

The higher is R, the sharper are the transmission resonances

$$\text{Finesse} = \frac{\text{width of the peak}}{\text{separation b/w two peaks}} = \pi \frac{r}{1-r^2} = \pi \frac{\sqrt{R}}{1-R}$$

(r is the ~~transmission~~ reflection coefficient for the amplitude, and $R = r^2$ — for intensity)

Finesse gives an estimate on the number of the round trips a photon can make inside the cavity before escaping (or being lost).