PHYS 404/690 Quantum and Nonlinear Optics

Problem set # 8 *(due April, 10)*

Each problem is 10 points. The problems marked with $*$ are required for graduate students only, and are extra credit problems for undergraduates.

P1 (Finish from the last homework) Consider a Mach-Zender interferometer and assume that one input mode is in a coherent state and that the other is in a squeezed vacuum state. Show that the phase fluctuations at the output can be reduced below the standard quantum limit ($\Delta\phi_{SQL} = 1/\sqrt{\bar{n}}$) to $\Delta\phi = e^{-r}/\sqrt{\bar{n}}$.

P2 When discussing the homodyne detection in class, we have assume that the local oscillator is a classical field (i.e. has no fluctuations). Show that even if we use a "proper" coherent state to describe the local oscillator, the homodyne detector output is going to be the same as before, if the local oscillator field is strong enough.

P3 Consider a pair state $|\eta\rangle$, which is defined as simultaneous eigenstate of the pair annihilation operator $\hat{a}\hat{b}$ with the eigenstate η (a complex number) and the number difference operator $\hat{a}^\dagger\hat{a} - \hat{b}^\dagger\hat{b}$ with zero eigenvalue. Find the description of the state $|\eta\rangle$ in terms of the number states, and examine its properties.

P4* In class, when discussing the homodyne detection, we (by default) assumed that a local oscillator and a quantum field are in the same spatial mode. What happens if their spatial modes are not exactly matched? How does the homodyne detector selects the mode it measures?