PHYS 404/690 Quantum and Nonlinear Optics

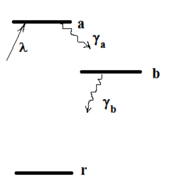
Problem set # 3 *(due January 13)*

Each problem is 10 points. The problems marked with $*$ are required for graduate students only, and are extra credit problems for undergraduates.

P1 Using the expression for susceptibility we obtain in class, calculate the refractive index of the atomic medium. Analyze the obtained expression for both unsaturated and saturated regime. Plot the graph of the refractive index vs laser detuning for a few different values of the saturation parameter.

P2 In class we have discussed a two-level system in which both levels decayed outside of the system, and both were independently repumped. Write down the block equations for the closed two-level system, in which the upper level decays only to the lower one with the rate Γ, and the lower level is the ground state. Find the steady-state solution for the population difference in this system. Is it possible to achieve the population inversion?

P3 Show that if a Fabri-Perot resonator is filled with an atomic medium with a susceptibility $\chi(\omega)$, the inter-mode frequency spacing is given by
$\omega_m - \omega_{m-1} = \frac{\pi c}{2n\ell\left(1+\frac{\omega}{2n^2}\frac{\partial\chi'}{\partial\omega}\right)}$,
where ℓ is the distance between the mirrors, n is the refractive index, and c is the speed of light in vacuum.



P4 Consider a three-level system shown. The reservoir level r is the ground-state. Upper level a decays into level b with the decay rate γ_a, and the lower level b decays into the ground state with the rate γ_b. An electric discharge effectively repopulates the state a at a rate λ. Write down the equations for populations of the levels a and b, and find the relationship between $\lambda, \gamma_a, \gamma_b$ that produces a steady-state population inversion.

P5* Consider again the closed two-level system, discussed in the second problem. Assume that at $t = 0$ the system was in the excited state. Calculate the population inversion in this system as a function of time.