**PHYS 404/690 Quantum and Nonlinear Optics**

***Problem set # 1*** *(due January 30)*

Each problem is 10 points. The problems marked with $*$ are required for graduate students only, and are extra credit problems for undergraduates.

**P1** Starting from the wave equation:
$-\nabla^2 \vec{E} + \frac{1}{c^2}\frac{\partial^2 \vec{E}}{\partial t^2} = -\mu_0 \frac{\partial^2 \vec{P}}{\partial t^2}$
derive the following equations of motion for the slowly-varying amplitude $E_0$ and phase $\phi$ , defined as $\vec{E} = \vec{e}_x E_0(z,t)\ e^{i[kz-\omega t+\phi(z,t)]}$:

$$\frac{\partial E_0}{\partial z} + \frac{1}{c}\frac{\partial E_0}{\partial t} = -\frac{k}{2\epsilon_0}\mathrm{Im}(\mathcal{P}_0)$$
$$E_0\left(\frac{\partial \phi}{\partial z} + \frac{1}{c}\frac{\partial \phi}{\partial t}\right) = \frac{k}{2\epsilon_0}\mathrm{Re}(\mathcal{P}_0)$$

**P2** Calculate the absorption length $(1/\alpha_0)$ for a 1.06 $\mu$m Nd:YAG laser beam propagating through a resonant linear medium with $10^{16}$ dipoles/$m^3$.

**P3** Calculate the magnitudes of the electric and magnetic fields for a 3 mW 628.3 nm laser focused down to a spot with a 2 $\mu$m radius. Assume constant intensity across the spot. How does this result scale with wavelength?

**P4** In an optical cavity, the resonant wavelengths are determined by the constructive-interference condition that an integer number of wavelengths must occur in a round trip. The corresponding frequencies are determined by these wavelengths and the speed of light in the cavity. Given a cavity with a medium having anomalous dispersion, would it be possible to have more than one frequency resonant for a single wavelength? How?

**P5**$^*$ Calculate the first and second-order coherence functions for the field:
$E^+(\vec{r},t) = \frac{E_0}{r}e^{-(\gamma+i\omega)(t-r/c)}\Theta(t-r/c)$,
where $\Theta$ is the Heaviside (step) function. This would be the field emitted by an atom located at $r=0$ and decaying spontaneously from time $t=0$, if such a field could be described totally classically.