Chapter 3

The Mössbauer Effect

3.1 Introduction

In 1957 Rudolf Mössbauer discovered the effect which bears his name. This was reported in 1958[**?**]. He was studying resonance fluorescence as a function of temperature. The process of resonance fluorescence is one in which γ-rays are resonantly scattered from nuclei. At low temperature this process was expected to turn off because the recoils given to the emitting nucleus and to the absorbing nucleus would cause the emitted γ-ray to not be at the right energy to resonantly interact with the absorbing nucleus. To see why this was expected, consider the case of ^{57}Fe, the nucleus for this lab. (Mössbauer studied ^{191}Ir, a much more difficult nucleus to study.) The pertinent information for ^{57}Fe are shown in Table 3.1. The 14.36 keV γ-ray gives a recoil momentum, P_r, to the emitting nucleus of 14.36 keV/c. This corresponds to an energy of recoil:

$$E_r = P_r^2/(2M(Fe)) = .018eV \tag{3.1}$$

The absorbing nucleus would also be given a similar amount. Thus there is about .036 eV energy mismatch. Next consider the width of the emission and absorbing lines:

$$\Gamma = \hbar/\tau = 4.7 \times 10^{-9} eV$$

So at low temperature the emission and absorption lines will not overlap and so resonant scattering should not occur. (At higher temperature or by the use of ultra-centrifuges, the energy mismatch can be compensated for.)

Mössbauer, however, observed an increase in scattering rather than the expected decrease at low temperature. The explanation of this surprising and Nobel prize winning discovery is that for some of the γ-rays the whole of the crystals in which the emitting and absorbing nuclei are embedded take up the recoil and thus the mass, M, in Eq. 3.1, is multiplied by a number on the order of Avogadro's number. This makes the recoil energy unmeasurably small.

	Ground State	First Excited State
Energy(keV)	0	14.36
Spin and parity	$\frac{1}{2}^-$	$\frac{3}{2}^-$
Magnetic Moment (nm)	0.0903	-0.153
Quadrupole moment (barns)	0	0.29
Mean lifetime (s)	Stable	1.4×10^{-7}
α (Int. Conv. Coef.)		9.7± 0.2

Table 3.1: Properties of ^{57}Fe

High Voltage	+1950 V
Dwell	100μS
CA =200	20 mm/s max

Table 3.2: Settings for the Mössbauer effect experiment.

3.2 Energy Splittings

The various shifts and splittings are shown inf Fig. 3.1.

3.2.1 Isomer Shift

The ismeric (or isomer) shift is the energy shift an atomic spectral line and gamma spectral lines due to replacement of one nuclear isomer by another. Isomers have the same atomic number Z.

$$E_a - E_e = \frac{2}{5}\pi Z e^2 [R_{is}^2 - R_{gr}^2][|\psi(0)_a|^2 - |\psi(0)_e|^2] \tag{3.2}$$

Here, $R_{is/gr}$ are the radii of the isomeric state and ground state respectively and $|\psi(0)_{a/e}|^2$ are the electron densities at the nucleus for the absorbing and emitting atoms. For a derivation see Wertheim[**?**]

3.2.2 Quadrupole Splitting

$$E = \frac{e^2 q Q}{4I(2I-1)}[3m^2 - I(I+1)] \tag{3.3}$$

where $q = \frac{1}{e}\partial^2 V/\partial z^2$, $Q = \frac{1}{e}\int \rho(3z^2 - r^2)d^3r$ and ρ is the nuclear charge density.

3.2.3 Zeeman Splitting

$$E = -\boldsymbol{\mu}\cdot\mathbf{H} = -g_I \mu_{nm} m H \tag{3.4}$$

Here μ is the nuclear moment and $\mathbf{H}$ is the magnetic field at the nucleus.

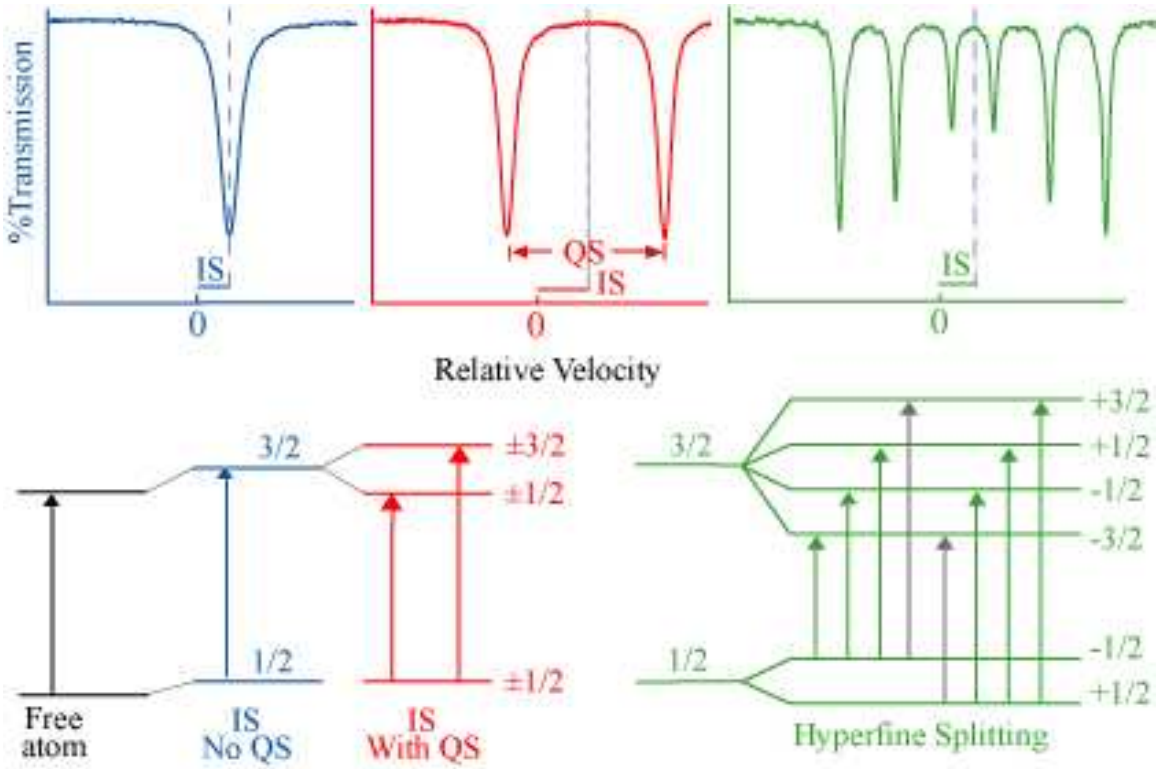

Figure 3.1: Absorption spectra for various characteristic shifts and splittings.

3.3 The Uncertainty Principle

The 14.4 keV excited state has an half-life of 98.3(3)ns. Thus its mean-life τ = 141.8(5)ns. The absorption cross-section

$$\sigma(E) = \sigma_0 \frac{(\Gamma_a/2)^2}{(E - E_0)^2 + (\Gamma_a/2)^2} \tag{3.5}$$

where $\Gamma_a = \hbar/\tau$ and σ_0 is:

$$\sigma_0 = 2\pi\lambda\!\!\!^{-2} \frac{2I_e + 1}{2I_g + 1} \frac{1}{1 + \alpha} \tag{3.6}$$

Since the energy distribution of the emitted photon has a similar shape, the expected cross-section is then the result of a convolution:

$$\sigma_{expt}(E) = \int_{-\infty}^{\infty} \omega(e)\sigma(E - e)de \tag{3.7}$$

where

$$\omega(e) = \frac{1}{\pi} \frac{\Gamma_e/2}{(e - E_0)^2 + (\Gamma_e/2)^2}$$

resulting in:

$$\sigma_{expt}(E) = \sigma_0 \cdot \frac{\frac{\Gamma_a}{2} \cdot \frac{\Gamma_a + \Gamma_e}{2}}{(E - E_0)^2 + \left(\frac{\Gamma_a + \Gamma_e}{2}\right)^2} \tag{3.8}$$

3.4 Apparatus

A block diagram of the apparatus is shown in Fig. 3.5. A ^{57}Co source is attached to the rod of the electro-mechanical drive. The drive signals can produce a **constant acceleration**. Following electron capture $^{57}_{27}Co \rightarrow electroncapture \rightarrow (^{57}_{26}Fe)^* \rightarrow (same)^{groundstate}$ (see Fig. 3.4) and transition to the 14.4 keV, $I = 3/2$ excited nuclear state, a γ-ray is emitted. The γ-ray traverses an absorber, and those that are not absorbed proceed to a proportional counter. This proportional counter has a thin Be window to allow the γ-ray through, is filled with a special gas, and has a small diameter central wire at high voltage. The entering γ-ray causes an atom to eject an electron with nearly the full γ-ray energy. This electron ionized further atoms. The electrons drift toward the central electrode and are accelerated to produce proportionally more electrons (hence the name). The charge is passed to a pre-amp. and amplifier which produce a voltage pulse proportional to the charge which is in turn proportional to the γ-ray energy. This pulse is sent to a computer controlled multi-channel analyzer (MCA) which makes a histogram of pulse heights.

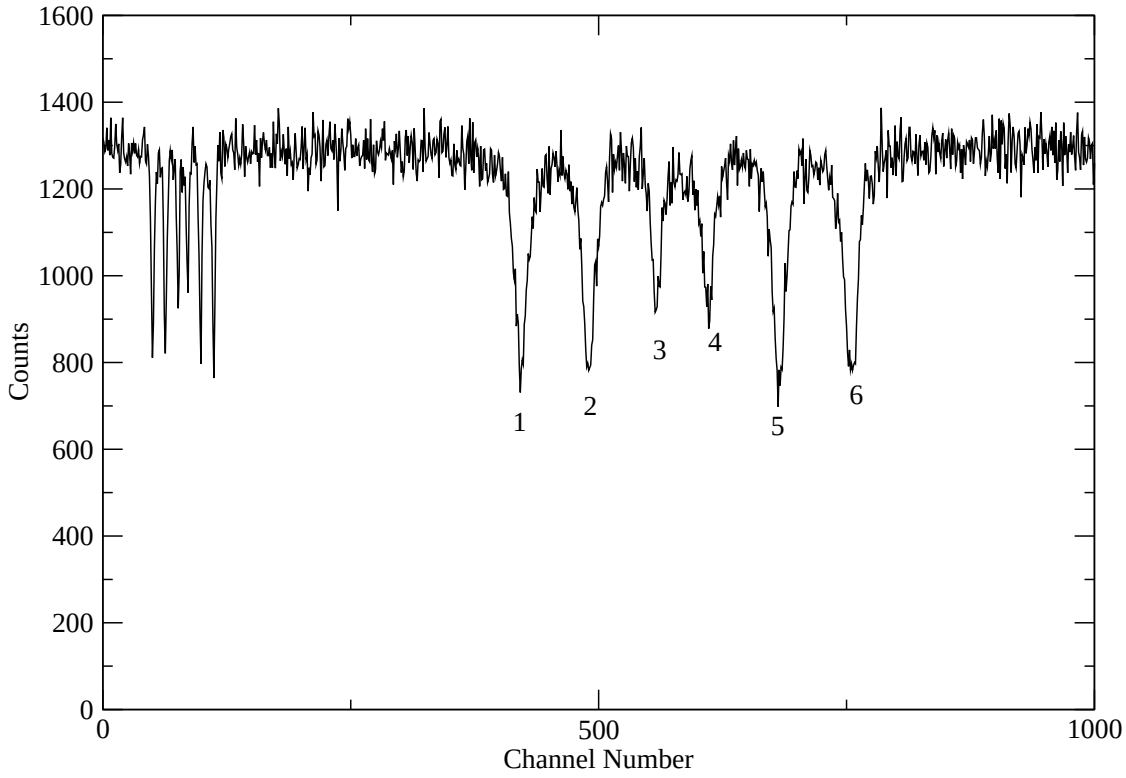

Figure 3.2: Spectrum observed with enriched ^{57}Fe absorber foil.

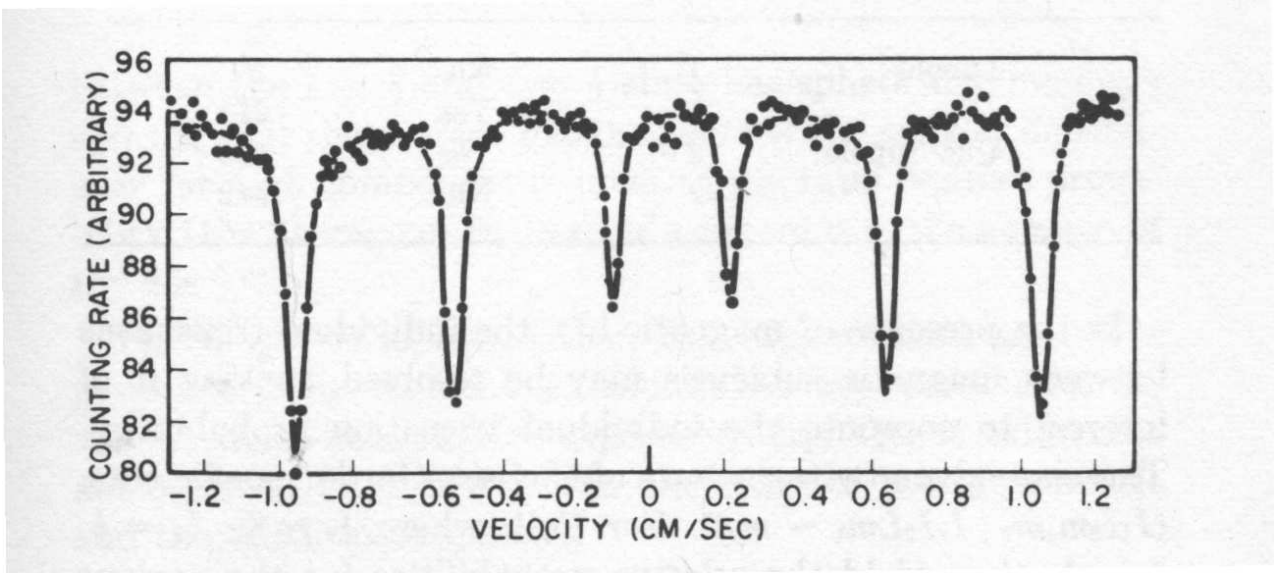

Figure 3.3: The spectrum observed by Wertheim[**?**]. This may be used to calibrate the current experiment.

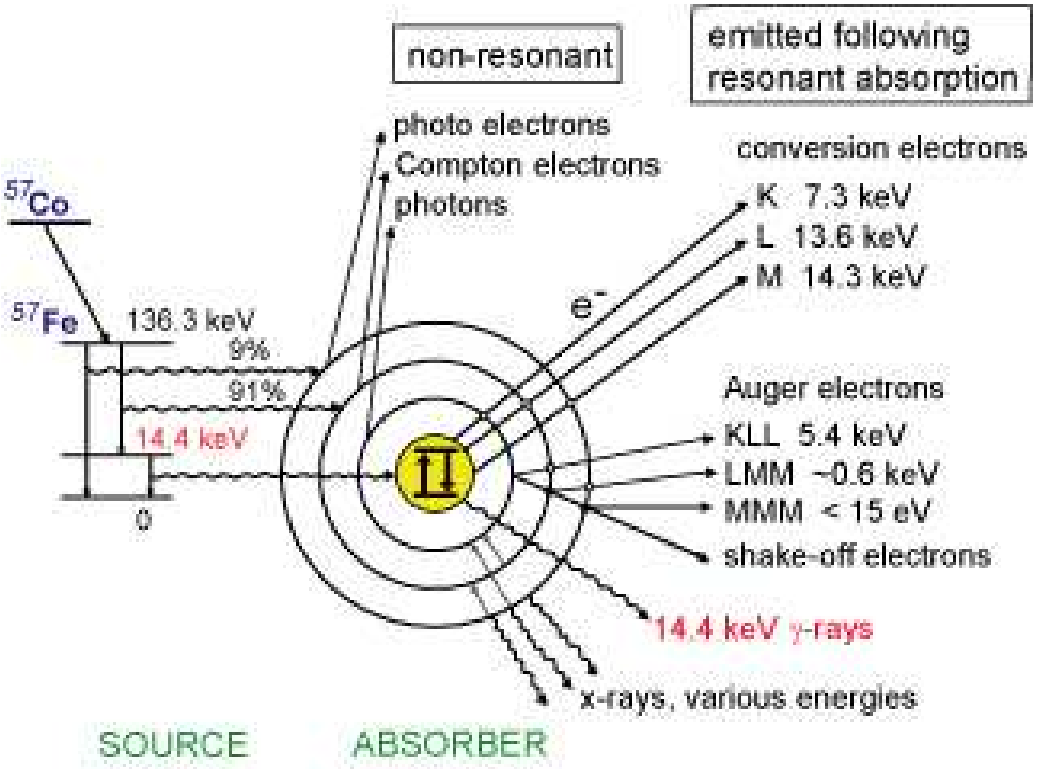

Figure 3.4: Decay and absorption scheme for ^{57}Fe. Taken from a web page of Dyar[**?**]. .

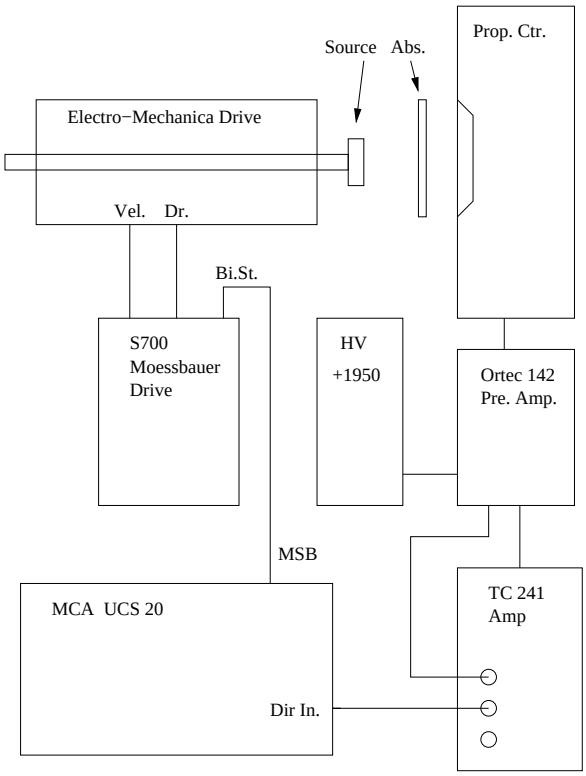

Figure 3.5: Block diagram of the Mössbauer apparatus.

3.5 Measurements

Obtain Mössbauer spectra for:

Enriched ^{57}Fe in Iron A pure Zeeman splitting case, i.e. no quadrupole splitting due to the symmetry of the site in iron.

Na-Prusside A pure quadrupole case. No magnetic field on the iron nucleus.

Stainless Steel Supposedly no splitting, but you will see an larger width for the single absorption dip.

3.5.1 Calibration

Calibrate the mechanical drive by using the measurements of Preston, Hanna, and Heberle**[?]** for the Zeeman splittings of ^{57}Fe in iron and your data. Preston et al. obtain the results shown in Table 3.3

m_g	m_e	Shift (mm/s)
-1/2	-3/2	-5.328
-1/2	-1/2	-3.084
-1/2	+1/2	-0.840
+1/2	-1/2	+0.840
+1/2	+1/2	+3.084
+1/2	+3/2	+5.328

Table 3.3: Calibration data.

3.5.2 Zeeman Field

Determine the magnetic field at the ^{57}Fe in iron for both the ground and excited states. (I know this is cyclical reasoning, since we have already used this information to determine the calibration.) Assume the nuclear moments are known. Compare this value with 330kG.

3.5.3 Quadrupole Splitting

Assuming that the quadrupole moment of the excited state is known, determine the electric field gradient for Na-Prusside. Determine the distance from an electronic charge which would give this electric field gradient.

3.5.4 Impurities

What value of local field variation would yield the absorption line-width you observe for stainless steel.

3.5.5 Uncertainty Principle

Fit the Na-Prusside absorption lines to Lorentzian line shapes':

$$d(i) = A - \frac{B}{(i-C)^2 + (D/2)^2} \tag{3.9}$$

where $d(i)$ is the counts in channel i. List your values for A, B, C, and D for both dips. From your calibration determine Γ_{expt}. Compare this to that expected for the lifetime of the 14.4 keV excited state.

Pound and Rebka[**?**] observed a $\Delta E/E$ of 1.13^{-12}. This was in their famous experiment to observe the gravitational red-shift of photons. How does your value for $\Delta E/E$ compare with theirs. If you equate a mass m with E_γ/c^2 what do you expect for the gravitational red shift for their 22.4 m tower?

3.6 Useful data

Isotopic abundance	2.14(1)
Ground State Properties	
I^P $1/2^-$	
μ	0.09062(3) nm
Excited State Properties	
I^P	$3/2^-$
E	14.412497(3) keV
E_r	1.95883310(4) 10-3 eV
α_{IC}	8.20
σ	2.56 10-18cm2
Q	0.21(2)b
$T_{1/2}$	98.3(3) ns
W	0.194(2) mm/s

Table 3.4: ^{57}Fe properties.