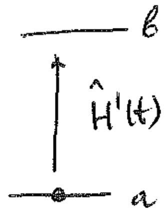
Sinusoidal perturbation



First-order perturbation

$$c_b(t) = -\frac{i}{\hbar}\int_0^t H'_{ba}(t')\, e^{i\omega_0 t'}\, dt'$$

$$H'_{ba}(t) = \langle \psi_b | \hat{H}'(t) | \psi_a \rangle$$

In most cases $\hat{H}'(t) = \hat{V}\cdot f(t)$ — time independent

$$H'_{ba}(t) = \langle \psi_b | \hat{V} | \psi_a \rangle \cdot f(t) = \underbrace{V_{ba}}_{\text{selection rules! and transition strength}} \cdot f(t)$$

$$c_b(t) = -\frac{i}{\hbar} V_{ba} \underbrace{\int_0^t f(t')\, e^{i\omega_0 t'}\, dt'}_{\text{similar to a fourier transform}}$$

If $f(t)$ is an oscillating function ...

$$f(t) = \cos\omega t = \frac{1}{2}\left(e^{i\omega t} + e^{-i\omega t}\right)$$

$$\int_0^t \cos\omega t\, e^{i\omega_0 t'}\, dt' = \frac{1}{2}\left\{\int_0^t e^{-i(\omega+\omega_0)t'}\, dt' + \int_0^t e^{i(\omega-\omega_0)t'}\, dt'\right\} =$$

$$= \frac{1}{2}\left\{\frac{e^{-i(\omega+\omega_0)t} - 1}{-i(\omega_0+\omega)} + \frac{e^{-i(\omega-\omega_0)t} - 1}{-i(\omega-\omega_0)}\right\}$$

~~If ω and ω_0 are~~

$$c_b(t) = +\frac{V_{ba}}{2\hbar}\,\frac{e^{i(\omega-\omega_0)t} - 1}{\omega-\omega_0} + \frac{V_{ba}}{2\hbar}\,\frac{e^{-i(\omega+\omega_0)t} - 1}{\omega+\omega_0}$$

If ω and ω_0 are not close to each other, both term needs to be accounted for, and the effect of the perturbation is usually small ($|c_b| \sim V_{ba}/2\hbar\omega_0$)

The resonance situation is much more interesting: $|\omega_0 - \omega| \ll \omega, \omega_0$

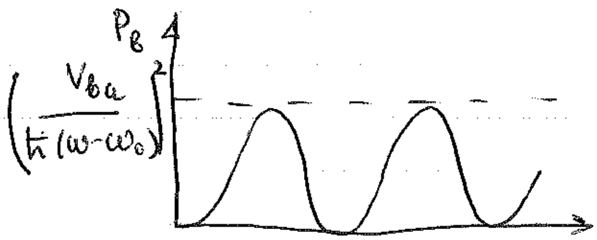
Since $|e^{i(\omega+\omega_0)t} - 1| \le 2$, and $|\omega-\omega_0| \ll \omega+\omega_0$,

the second term in $c_b(t)$ is negligibly small

$$c_b(t) \simeq + \frac{V_{ba}}{2\hbar} \frac{e^{i(\omega-\omega_0)t} - 1}{\omega-\omega_0} = + \frac{V_{ba}}{2\hbar} e^{\frac{i(\omega-\omega_0)t}{2}} \times$$

$$\times \frac{e^{i(\omega-\omega_0)t/2} - e^{-i(\omega-\omega_0)t/2}}{\omega-\omega_0} = + \frac{iV_{ba}}{\hbar} e^{\frac{i(\omega-\omega_0)t}{2}} \frac{\sin\frac{\omega-\omega_0}{2}t}{\omega-\omega_0}$$

$$P_b(t) = |c_b(t)|^2 = \left|\frac{V_{ba}}{\hbar}\right|^2 \frac{\sin^2\frac{\omega-\omega_0}{2}t}{(\omega-\omega_0)^2} = \left|\frac{V_{ba}}{\hbar}\right|^2 \frac{(1-\cos(\omega-\omega_0)t)}{2(\omega-\omega_0)^2}$$



Period of oscillations $= 2\pi/|\omega-\omega_0|$

The closer we are to the resonance $\omega = \omega_0$, the larger is the transition probability

$$P_b^{(max)} = \frac{V_{ba}}{\hbar(\omega-\omega_0)}$$

Can it become more than one?

No $\rightarrow$ that breaks the perturbation approximation we made; that c_a^* is basically unchanged.

However, we can solve the problem exactly in the resonant case

Resonant excitation - exact solution

$H'_{ba} = \frac{1}{2} V_{ba} e^{-i\omega t}$ $\quad$ $H'_{ab} = (H'_{ba})^* = \frac{1}{2} V_{ba} e^{i\omega t}$

(rotation wave approximation)

Here we already neglect the second complex exponent that is going to give ~~a~~ us $e^{\pm i(\omega + \omega_0)t}$ terms, as we know their contribution is going to be negligible.

$$\begin{cases} \dot{c}_a = -\frac{i}{\hbar} H'_{ab}(t) e^{-i\omega_0 t} c_b = -\frac{i V_{ba}}{2\hbar} e^{i(\omega - \omega_0)t} c_b \\ \dot{c}_b = -\frac{i}{\hbar} H'_{ba}(t) e^{i\omega_0 t} c_a = -\frac{i V_{ba}}{2\hbar} e^{-i(\omega-\omega_0)t} c_a \end{cases}$$

There is an exact solution for $\omega - \omega_0 \neq 0$, but to avoid cumbersome math, we are going to just solve for $\omega = \omega_0$

$$\begin{cases} \dot{c}_a = -\frac{i V_{ba}}{2\hbar} c_b \\ \dot{c}_b = -\frac{i V_{ba}}{2\hbar} c_a \end{cases} \Rightarrow \ddot{c}_a = -\frac{i V_{ba}}{2\hbar} \dot{c}_b = -\left(\frac{V_{ba}}{2\hbar}\right)^2 c_a$$

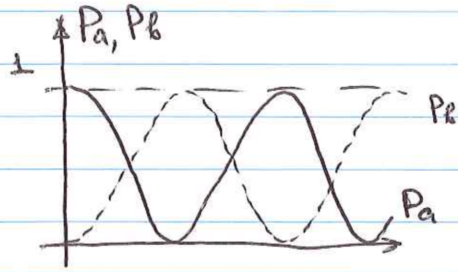
(same for c_b)

This is an equation for a harmonic oscillator:

$\ddot{x} + \Omega^2 x = 0$ $\quad$ $\Omega = V_{ba}/2\hbar$

$c_a(t) = \cos \Omega t$ $\quad$ (since $c_a(t=0) = 1$)

$c_b(t) = \sin \Omega t$ $\quad$ (since $c_b(t=0) = 0$)



$P_a(t) = \cos^2 \Omega t$

$P_b(t) = \sin^2 \Omega t$

Rabi oscillations

Rabi frequency $\Omega_R = V_{ba}/2\hbar$

In general, the population of the state b oscillates at the generalized Rabi frequency

$$\tilde{\Omega}_R = \frac{1}{2}\sqrt{(\omega - \omega_0)^2 + \left(\frac{V_{ba}}{\hbar}\right)^2}$$

Why Rabi oscillations (aka Rabi flopping) are important? They are a key to quantum control.

By choosing the duration of the field (often either optical or microwave or radio frequency electromagnetic field) one can ~~create~~ transfer ~~an~~ ~~ato~~ a particle from one well-defined state to another, or to create a well-defined quantum superposition of the two states.

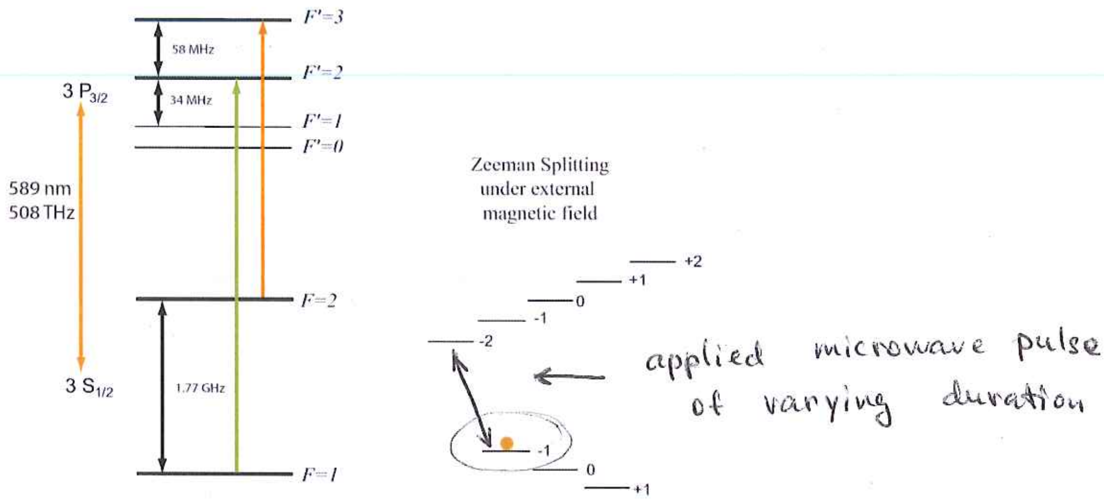


Figure 3.6: Sodium D_2 transition hyperfine structure : The $3S_{1/2}$ to $3P_{3/2}$ levels are shown. Most of the cooling and probing is performed on the cycling transition, $F = 2$ to $F' = 3$, shown by the orange arrow. The repumping transition, $F = 1$ to $F' = 2$, is shown with a green arrow. An externally applied magnetic field breaks the degeneracies of the hyperfine ground states, with sublevels shown on the right. Magnetic trapping and evaporative cooling is performed in the $F = 1$, $m_F = -1$ state (indicated above).

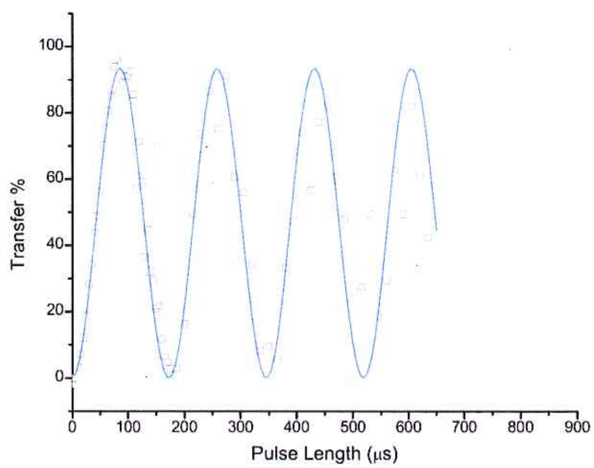


Figure 3.16: Microwave Rabi flopping: A microwave field is turned on for a certain pulse duration, shown on the horizontal axis. The atoms are initially in the $F = 1$, $m_F = -1$ state. The microwave pulse transfers atoms to the $F = 2$, $m_F = -2$ state. At the end of the pulse, we measure the populations in each of the states, and determine the transfer % to the $F = 2$, $m_F = -2$ state. On tuning the microwave frequency to the resonance condition for the $F = 1$, $m_F = -1$ and the $F = 2$, $m_F = -2$ states, we observe Rabi flopping where the population oscillates between the two states. The line is a least squares fit to the initial 2 oscillations. Over long times (>200 μs), fluctuations in the magnetic bias field dephase the oscillations.

[source of images]

ABSTRACT

Title of dissertation: A RING WITH A SPIN: SUPERFLUIDITY IN A TOROIDAL BOSE-EINSTEIN CONDENSATE

Anand Krishnan Ramanathan, Doctor of Philosophy, 2011

Dissertation directed by: Steve Rolston
Chemical Physics Program

Superfluidity is a remarkable phenomenon. Superfluidity was initially characterized by flow without friction, first seen in liquid helium in 1938, and has been studied extensively since. Superfluidity is believed to be related to, but not identical to Bose-Einstein condensation, a statistical mechanical phenomena predicted by Albert Einstein in 1924 based on the statistics of Satyendra Nath Bose, where bosonic atoms make a phase transition to form a Bose-Einstein condensate (BEC), a gas which has macroscopic occupation of a single quantum state.

Developments in laser cooling of neutral atoms and the subsequent realization of Bose-Einstein condensates in ultracold gases have opened a new window into the study of superfluidity and its relation to Bose-Einstein condensation. In our atomic sodium BEC experiment, we studied superfluidity and dissipationless flow in an all-optical toroidal trap, constructed using the combination of a horizontal