Born series

General 'solution'

$$\psi(\vec{r}) = \psi_0(\vec{r}) + \int G(\vec{r}-\vec{r}\,')\, U(\vec{r}\,')\, \psi(\vec{r}\,')\, d^3\vec{r}\,'$$

If the scattering is a weak perturbation

Zeroth approximation: $U(\vec{r}\,') = 0$, $\psi^{(0)}(\vec{r}) = \psi_0(\vec{r})$

First approximation $\psi^{(1)}(\vec{r}) = \psi_0(\vec{r}) + \int G(\vec{r}-\vec{r}\,')U(\vec{r}\,')\psi^{(0)}(\vec{r}\,')d^3r'$
(first Born approximation)

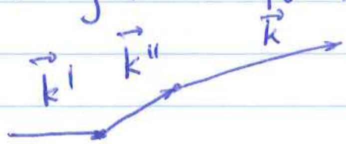
$$f^{(1)}(\theta) = -\frac{m}{2\pi\hbar^2}\int e^{i(\vec{k}'-\vec{k})\vec{r}\,'}\, U(\vec{r}\,')\, d^3\vec{r}\,'$$

Similar to single scattering event

Second approximation

$$\psi^{(2)}(\vec{r}) = \psi_0(\vec{r}) + \int G(\vec{r}-\vec{r}\,')U(\vec{r}\,')\psi^{(1)}(\vec{r}\,')\, d^3\vec{r}\,' =$$

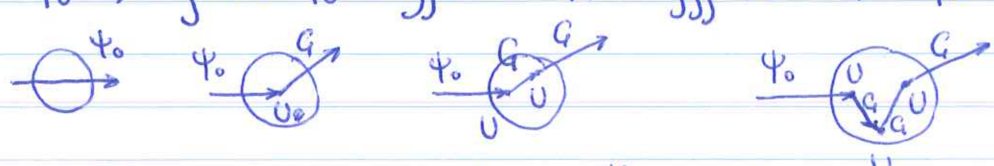
$$= \psi_0(\vec{r}) + \int G(\vec{r}-\vec{r}\,')\psi_0(\vec{r}\,')U(\vec{r}\,')d^3\vec{r}\,' + \int G(\vec{r}-\vec{r}\,')U(\vec{r}\,')d^3\vec{r}\,' \int G(\vec{r}\,'-\vec{r}\,'')U(\vec{r}\,'')\psi_0(\vec{r}\,'')\, d^3\vec{r}\,''$$



two scattering "events"
$\vec{k}' \longrightarrow \vec{k}'' \longrightarrow \vec{k}$

Following the same procedure, we can construct a series

$$\psi(\vec{r}) = \psi_0(\vec{r}) + \int G \cdot U \cdot \psi_0 + \iint GUGU\psi_0 + \iiint GUGUGU \cdot \psi_0 + \dots$$



$G \rightarrow$ propagator, tells you how the disturbance propagates
$U_0 \rightarrow$ vertex factor (strength of interaction)

This approach is very similar to higher-order time-dependent perturbation theory

Remember, for a two-level system we have

$$\dot{c}_a = -\frac{i}{\hbar} H'_{ab} e^{-i\omega_0 t} c_b \qquad \dot{c}_b = -\frac{i}{\hbar} H'_{ba} e^{i\omega_0 t} c_a$$

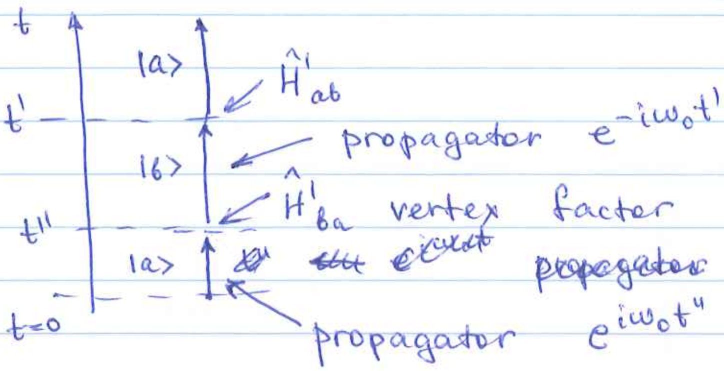
Solving by a power series of H_{ab}

"0" $\quad c_a = 1 \quad c_b = 0 \quad$ (no change)

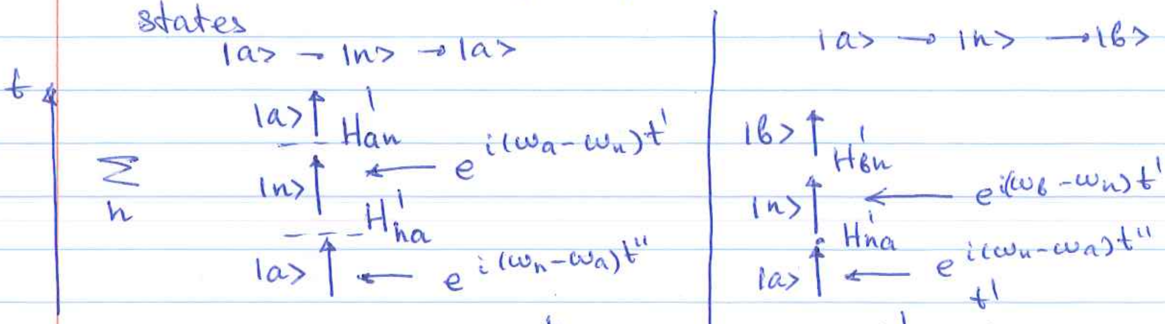
"1" $\quad c_b^{(1)} = -\frac{i}{\hbar}\int H'_{ba}(t') e^{i\omega_0 t} dt' \qquad c_a^{(1)} = 1$ (no change)

"2" $\quad \dot{c}_a^{(2)} = -\frac{i}{\hbar} H'_{ab}(t) e^{-i\omega_0 t} c_b^{(1)} = +\left(\frac{i}{\hbar}\right)^2 H'_{ab}(t) e^{-i\omega_0 t} \int H'_{ba}(t') e^{i\omega_0 t'} dt'$

$$c_a^{(2)} = 1 + \left(\frac{i}{\hbar}\right)^2 \int_0^t H'_{ab}(t') e^{-i\omega_0 t'} dt' \int_0^{t'} H'_{ba}(t'') e^{i\omega_0 t''} dt''$$



Now we can possibly extend it to multiple states



$$c_a^{(2)} = 1 + \left(\frac{i}{\hbar}\right)^2 \sum_n \int_0^t H'_{an}(t') e^{+i(\omega_a-\omega_n)t'} dt' \int_0^{t'} H'_{na}(t'') e^{i(\omega_n-\omega_a)t''} dt''$$

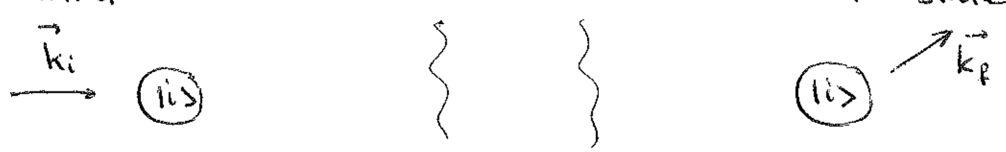
$$c_b^{(2)} = \left(\frac{i}{\hbar}\right)^2 \sum_n \int_0^t H'_{bn}(t') e^{i(\omega_b-\omega_n)t'} dt' \int_0^{t'} H'_{na}(t'') e^{i(\omega_n-\omega_a)t''} dt''$$

In principle, one can often draw Fynman diagrams to represent a power series of corrections, created in the perturbation theories. However, the main strength of the Fynmann diagrams for QED is that they provide a streight forward connection b/w intuitive classical description of possible processes, and their strict mathematical quantum ~~[illegible]~~ calculations. In some other situations the diagrams can be drawn from mathematical expressions, but may not be as helpful on its own.

Example: photon scattering on an atomic electron

This is a two-photon event; as it must contain one act of absorption of a photon, with an atom changing its state, and one act of emission (with atomic state change again)

Initial state — final state



& Light - atom interaction

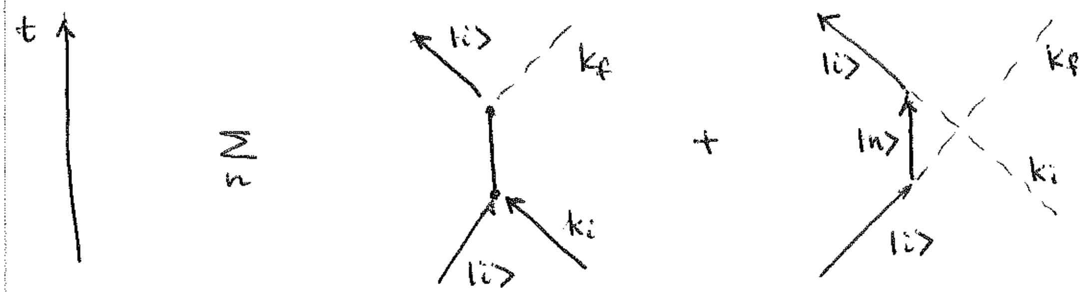
—— $|n\rangle$ absorption $|n\rangle\langle i|\hat{a}_{k_i}$ $E_n > E_i$

—— $|i\rangle$ emission $|i\rangle\langle n|\hat{a}^{+}_{k_f}$

—— $|i\rangle$ $|n\rangle\langle i|\hat{a}^{+}_{k_f}$ $E_n < E_i$

—— $|n\rangle$ $|i\rangle\langle n|\hat{a}_{k_i}$

Possible scenarios



intermediat state : no photons 2 photons

Amplitude for each process $$M_2^{\pm} = \frac{H_{in} H_{ni}}{E_i - E_n \pm \hbar\omega}$$

Total amplitude $$M_{total} = \underbrace{\sum_n M_2^{+}}_{\text{all intermediate states, first diagram}} + \underbrace{\sum_n M_2^{-}}_{\text{all intermediate state, second diagram}}$$

Transition rate $\propto \frac{2\pi}{\hbar} |M_{total}|^2$