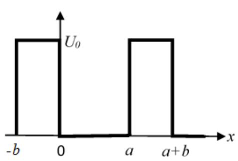
PHYS 314
Problem set # 7 *(due April 1)*
Each problem is 10 points, unless stated otherwise.

Griffiths: 5.34, 5.35

Q1 Suppose that we have N identical bosons of mass m in a one dimensional box of length $L = Na$ $(0 < x < Na)$, with the Hamiltonian
$\hat{H} = \sum_{i=1}^{N} \hat{p}_i^2/2m + \lambda/2 \sum_{i \neq j} \delta(x_i - x_j)$.
(a) Suppose $\lambda = 0$. What is the ground state wave function? Call your answer $\Psi_0(x_1, \cdots x_N)$. What is the energy of this state $E_0(\lambda = 0)$? How does it scales with the number of particles if $N \gg 1$?
(b) Now, let us suppose that $\lambda > 0$, so that these bosons repel each other. Calculate the first-order correction to the ground-state energy $\Delta E_0(\lambda)$ and comment on your answer. *Hint: note, that each individual interaction term only depends on two coordinates, and thus when integrating over N-dimensional wave function, we can immediately integrate over $N-2$ coordinates and get factors of unity.*
(c) Now we throw out the bosons, and put N fermions with the same mass in the same box. What is the ground state $\tilde{\Psi}_0(x_1, \cdots x_N)$ for the fermionic system, if $\lambda = 0$? *Hint: it may be convenient to use* ***Slater determinant*** *to write this wave function.*
(d) What is the energy of this state $\tilde{E}_0(\lambda = 0)$? Calculate it in the limit of large particle number $N \gg 1$ and argue why it must be independent of λ.

Q2 Consider a 1D model of a crystal in which a periodic potential is shown in the figure.



Assume that a particle of mass m is moving in this potential with the total energy $E > U_0$. Using the conditions for the wave function continuity and the Block theorem $\phi(x) = e^{ik(a+b)}\psi(x - a - b)$, write four equations for boundary conditions at $x = 0$ and $x = a$ that, if solved can provide the indirect relation between the lattice parameter k and the energy of the particle E.
For the brave at heart (and 4 points of extra credit): show that this relation is:
$\cos k(a+b) = \cos k_1 a \cos k_2 a - \frac{1}{2}\left(\frac{k_1}{k_2} + \frac{k_2}{k_1}\right) \sin k_1 a \sin k_2 b$,
where $k_1 = \sqrt{2mE}/\hbar$ and $k_1 = \sqrt{2m(E - U_0)}/\hbar$.

Q3 An experimentalist has carefully prepared a particle of mass m in the first excited state of a one-dimensional harmonic oscillator of frequency ω, when she sneezes and knocks the center of the potential well a small distance A to one side. It takes her time T to notice this, after which she immediately puts the center back where it was. Find, to lowest order in A, the probabilities P_0 and P_2 that the oscillator will now be in its ground state and its second excited state after the ordeal.