PHYS 314
***Problem set # 2** (due February 4)*
Each problem is 10 points, unless stated otherwise.

Griffiths, Ch. 6: 6.29, 6.37(20 points)
The following integral may be handy for the problem 6.29:

$$\int_0^\alpha x^n e^{-x} dx = n! \left(1 - e^{-\alpha} \sum_{k=0}^{n} \frac{\alpha^k}{k!}\right)$$

Q1 A particle with charge q and mass m is in ground energy state inside a one-dimensional infinite square well with width $2a$, centeredat $x = 0$. Find the first- and second-order corrections of the ground-level energy, if this particle is placed inside a uniform electric field E. (The answers may include a dimensionless summation, if necessary)

Q2 Consider a spinless particle of mass μ and charge q under the simultaneous influence of a uniform magnetic field and electric fields. The interaction Hamiltonian of the the two terms are:
$\hat{H}_M = -\frac{q}{2\mu c}\vec{B} \cdot \vec{L}$; $\hat{H}_E = -q\vec{E} \cdot \vec{r}$.
Show that
$|\langle \ell\, m|\hat{H}_M + \hat{H}_E|\ell'\, m'\rangle|^2 = |\langle \ell\, m|\hat{H}_M|\ell'\, m'\rangle|^2 + |\langle \ell\, m|\hat{H}_E|\ell'\, m'\rangle|^2$,
and that, always, one of the matrix elements $\langle \ell\, m|\hat{H}_M|\ell'\, m'\rangle$ or $\langle \ell\, m|\hat{H}_E|\ell'\, m'\rangle$ vanishes.
Helpful relationships: For the magnetic field calculations: $\hat{L}_\pm = \hat{L}_x \pm i\hat{L}_y$, $\langle \ell\, m|\hat{L}_\pm|\ell'\, m'\rangle \propto \delta_{\ell\ell'}\delta_{mm'\pm1}$, $\langle \ell\, m|\hat{L}_z|\ell'\, m'\rangle \propto \delta_{\ell\ell'}\delta_{mm'}$.
For electric field calculations: it is possible to express the coordinates using the spherical functions: $x \propto r(Y_{11} - Y_{1-1})$, $y \propto r(Y_{11} + Y_{1-1})$, and $z \propto r(Y_{10}$. Then apply the following rule: $\int Y^*_{\ell_1 m_1} Y_{\ell_2 m_2} Y_{\ell_3 m_3} \sin\theta d\theta d\phi \neq 0$ only if $m_1 + m_2 + m_3 = 0$, $\ell_1 + \ell_2 + \ell_3$ is even, and $|\ell_1 - \ell_2| \leq \ell_3 \leq |\ell_1 + \ell_2|$.