Two-level system evolution

1. Spin 1/2 particle in $\vec{B} = \{0, 0, B_z\}$ magnetic field

$$\hat{H}|+z\rangle = \frac{1}{2}\hbar\omega_0 \; |+z\rangle \qquad \hat{H}|-z\rangle = -\frac{1}{2}\hbar\omega_0 \; |-z\rangle$$

$$|+x\rangle(t) = \frac{1}{\sqrt{2}}\left(e^{-i\omega_0 t/2} |+z\rangle + e^{i\omega_0 t/2} |-z\rangle \right)$$

$$\langle S_z\rangle = 0 \qquad \langle S_x\rangle(t) = \frac{\hbar}{2}\cos\omega_0 t \qquad \langle S_y\rangle(t) = \frac{\hbar}{2}\sin\omega_0 t$$

2. Neutrino oscillations

three types of ν: $\nu_e \quad \nu_\mu \quad \boxed{\nu_\tau}$ do not include for simplicity

three masses $m_1 \quad m_2 \quad \boxed{m_3}$

Neutrino flavors don't match precisele the masses!

mass ≡ energy eigenstates $|1,2\rangle$

$$E_{1,2} = \sqrt{p^2c^2 + m_{1,2}^2 c^4} \approx pc\left(1 + \frac{m_{1,2}^2 c^2}{2p^2}\right)$$

p – momentum, fixed

$$\hat{H}|1,2\rangle = E_{1,2}|1,2\rangle \qquad |1\rangle \to e^{-iE_1 t/\hbar}|1\rangle$$

$$|2\rangle \to e^{-iE_2 t/\hbar}|2\rangle$$

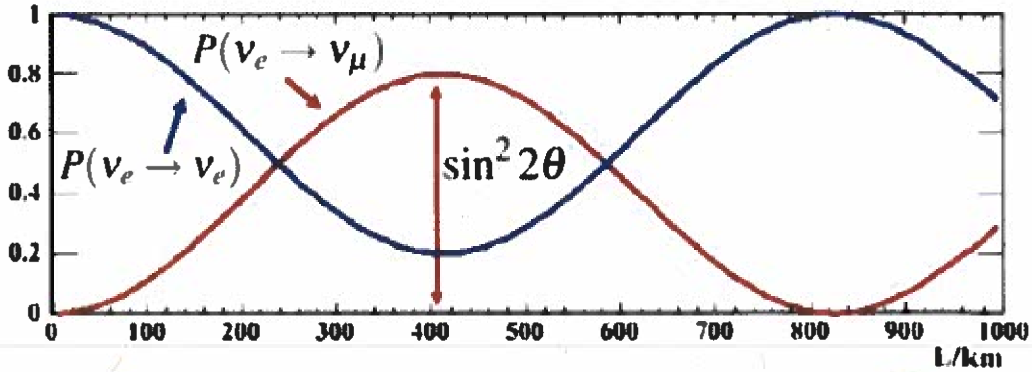
$$|\nu_e\rangle = \cos\theta\,|1\rangle - \sin\theta\,|2\rangle$$

$$|\nu_\mu\rangle = \sin\theta\,|1\rangle + \cos\theta\,|2\rangle \quad \Rightarrow \text{evolve in time!}$$

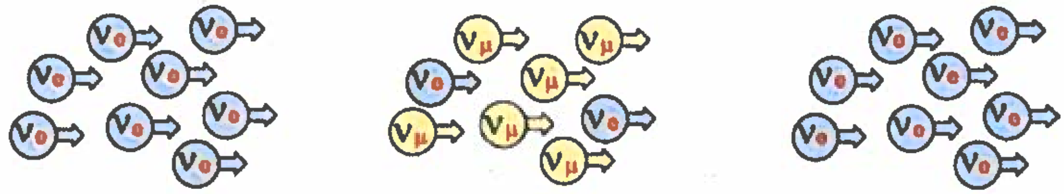
$$P(\nu_e \to \nu_e) = 1 - \sin^2 2\theta \, \sin^2\left(\Delta m^2 c^4 \frac{L}{4E\hbar c}\right); \quad \Delta m^2 = m_1^2 - m_2^2$$

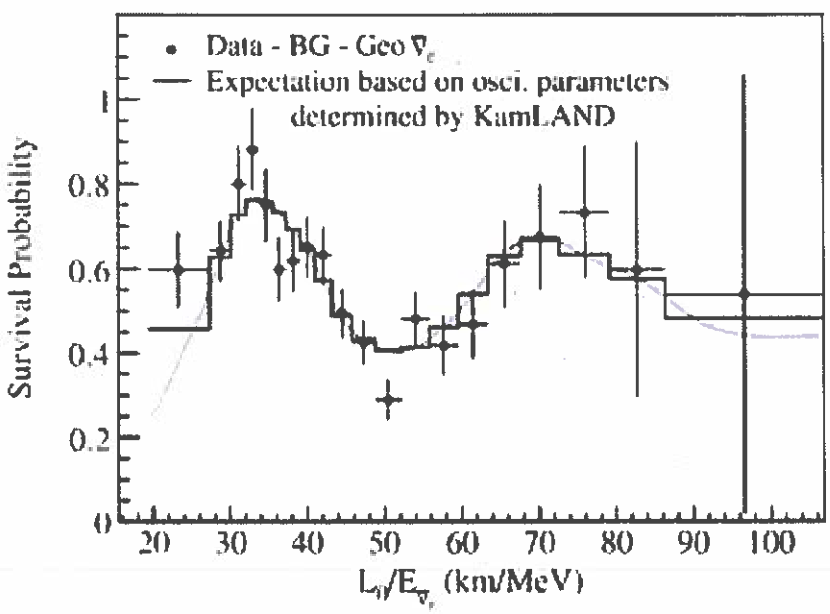
Neutrino oscillations

•e.g. $\Delta m^2 = 0.003\,\mathrm{eV}^2, \quad \sin^2 2\theta = 0.8, \quad E_\nu = 1\,\mathrm{GeV}$



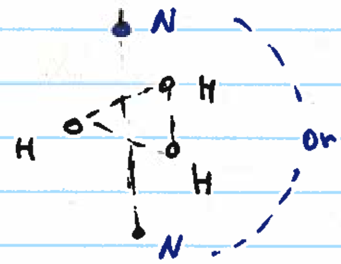
$= c \cdot t$





3. Ammonia molecules

Two orientations NH_3



$\downarrow \vec{\mu}$ $|1\rangle$

or

$\uparrow \vec{\mu}$ $|2\rangle$

geometrical states

Hamiltonian: $\hat{H} = \begin{pmatrix} E_0 & -A \\ -A & E_0 \end{pmatrix}$

$-A$ characterises the energy required to "flip" the N-atom position

Eigenvalues of $\hat{H}$: $\det \begin{pmatrix} E_0-\lambda & -A \\ -A & E_0-\lambda \end{pmatrix} = 0$

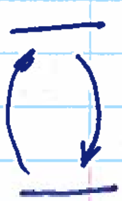
$(E_0-\lambda)^2 - A^2 = 0 \qquad \lambda_\pm = E_0 \pm A$

Eigen states $\quad H|\pm\rangle = \lambda_\pm |\pm\rangle$

$|\pm\rangle = \begin{pmatrix} c_1 \\ c_2 \end{pmatrix} \qquad \lambda_+ = E_0 + A \quad \begin{pmatrix} -A & -A \\ -A & -A \end{pmatrix}\begin{pmatrix} c_1 \\ c_2 \end{pmatrix} = 0$

$\lambda_- = E_0 - A \quad \begin{pmatrix} A & -A \\ -A & A \end{pmatrix}\begin{pmatrix} c_1 \\ c_2 \end{pmatrix} = 0 \qquad c_1 = -c_2$

$c_1 = c_2$



$E_+ = E_0 + A \qquad |+\rangle = \frac{1}{\sqrt{2}}(|1\rangle + 2\rangle)$ Symmetric syperposition

$E_- = E_0 + A \qquad |-\rangle = \frac{1}{\sqrt{2}}(|1\rangle + |2\rangle)$ anti-symmetric syperposition

$|1\rangle = \frac{1}{\sqrt{2}}|+\rangle + \frac{1}{\sqrt{2}}|-\rangle \longrightarrow \frac{1}{\sqrt{2}} e^{-i(E_0+A)t/\hbar}|+\rangle + \frac{1}{\sqrt{2}} e^{-i(E_0-A)t/\hbar}|-\rangle$

$$|1\rangle(t) = \frac{1}{\sqrt{2}} e^{-iE_0t/\hbar} \left[e^{-iAt/\hbar} |+\rangle + e^{iAt/\hbar} |-\rangle \right] =$$

$$= \frac{1}{\sqrt{2}} e^{-iE_0t/\hbar} \left[\left(\cos\frac{At}{\hbar} - i\sin\frac{At}{\hbar}\right)|+\rangle + \left(\cos\frac{At}{\hbar} + i\sin\frac{At}{\hbar}\right)|-\rangle \right]$$

$$= e^{-iE_0t/\hbar} \left[\cos\frac{At}{\hbar} |1\rangle + \frac{1}{i} \sin\frac{At}{\hbar} |2\rangle \right]$$

If we placed a NH_3 molecule in one position, it is going to flip back and force with frequency $\nu_{NH_3} = 2A/h = 24\,GHz$

The basis for NH_3 maser operation

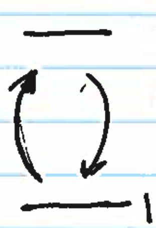
In reality, we can much more efficiently flip the system b/w two states using resonant electric or magnetic field oscillating on the frequency of this transition

Back to spins: strong constant B_0 along z and rf field $B_1 \cos\omega t$ along x

The off-diagonal term in the hamiltonian is proportional to B_1

$$\hat{H} = \frac{\hbar}{2} \begin{pmatrix} \omega_0 & \underline{\omega_1 \cos\omega t} \\ \underline{\omega_1 \cos\omega_1} & -\omega_0 \end{pmatrix}$$

induced transitions, the highest p rate if $\omega = \omega_0$

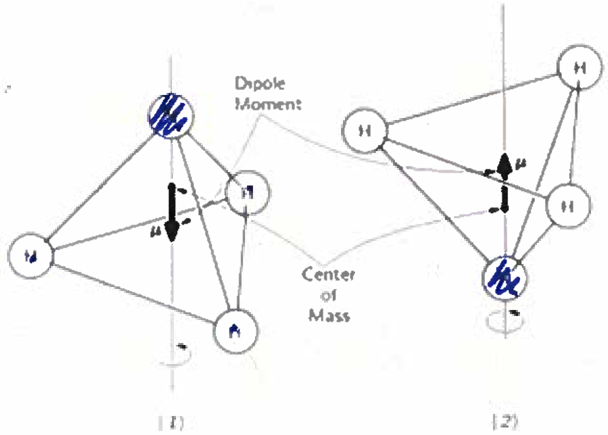


$|+z\rangle$ $E_+ = \hbar\omega_0/2$

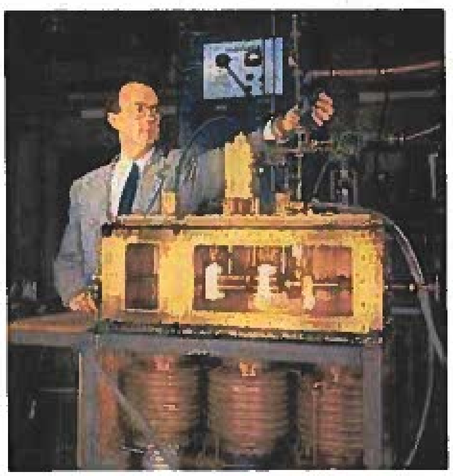
$|-z\rangle$ $E_- = -\hbar\omega_0/2$

$$P_{-z}(t) = \cos^2 \frac{\omega_1 t}{4} \qquad P_{+z}(t) = \sin^2 \frac{\omega_1 t}{4}$$

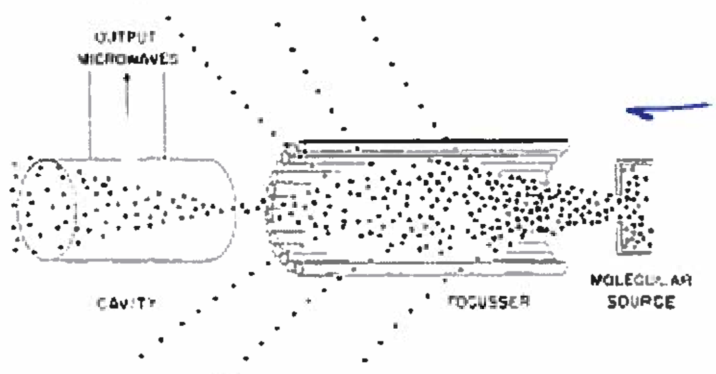
NH_3



Two equally possible orientations



NH_3 maser



Microwave
A amplification via
Stimulated
Emission of
Radiation