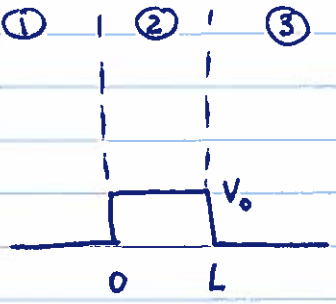
Interference with quantum waves



What is the transmission probability for the particle?

If $k = \sqrt{\frac{2mE}{\hbar^2}}$ and $k_1 = \sqrt{\frac{2m(E-V_0)}{\hbar^2}}$

① $\psi_I = \underbrace{A e^{ik_1 x}}_{\text{incident}} + \underbrace{B e^{-ikx}}_{\text{reflected}}$

② $\psi_{II} = C e^{ik_1 x} + D e^{-ik_1 x}$ (back & forth b/w the walls)

③ $\psi_{III} = F e^{ik(x-L)}$ transmitted

Boundary conditions

	$x = 0$	$x = L$
continuity	$A+B = C+D$	$Ce^{ik_1 L} + De^{-ik_1 L} = F$
smoothness	$ik(A-B) = ik_1(C-D)$	$ik_1(Ce^{ik_1 L} - De^{-ik_1 L}) = ikF$

For transmission I need to find $\left|\frac{F}{A}\right|^2$

$$2A = C\left(1+\frac{k_1}{k}\right) + D\left(1-\frac{k_1}{k}\right)$$

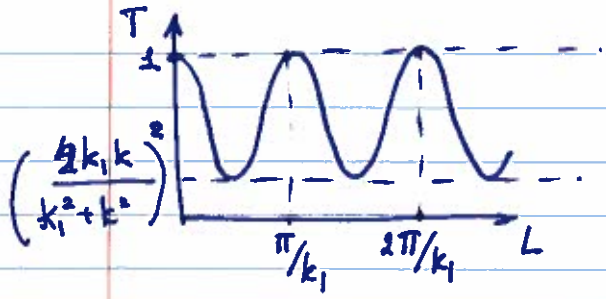
$$2Ce^{ik_1 L} = (1+k/k_1)F$$
$$2De^{-ik_1 L} = (1-k/k_1)F$$

$$2A = \underbrace{\left(1+\frac{k_1}{k}\right)\left(1+\frac{k}{k_1}\right)}_{2+\frac{k_1^2+k^2}{k_1 k}}\frac{1}{2}e^{-ik_1 L}F + \underbrace{\left(1-\frac{k_1}{k}\right)\left(1-\frac{k}{k_1}\right)}_{2-\frac{k_1^2+k^2}{k_1 k}}\frac{1}{2}e^{ik_1 L}F$$

$$2A = \left[\frac{1}{2}e^{-ik_1 L}\left(2+\frac{k_1^2+k^2}{k_1 k}\right) + \frac{1}{2}e^{ik_1 L}\left(2-\frac{k_1^2+k^2}{k_1 k}\right)\right]F$$

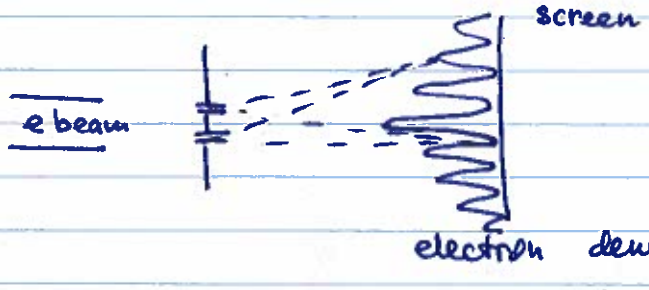
$$A = \left(\cos k_1 L + \frac{k_1^2+k^2}{2ik_1 k}\sin k_1 L\right)F$$

$$T = \left|\frac{F}{A}\right|^2 = \frac{1}{\cos^2 k_1 L + \left(\frac{k_1^2+k^2}{2k_1 k}\right)^2 \sin^2 k_1 L} = \frac{1}{1+\left(\frac{k_1^2-k^2}{2k_1 k}\right)^2 \sin^2 k_1 L}$$



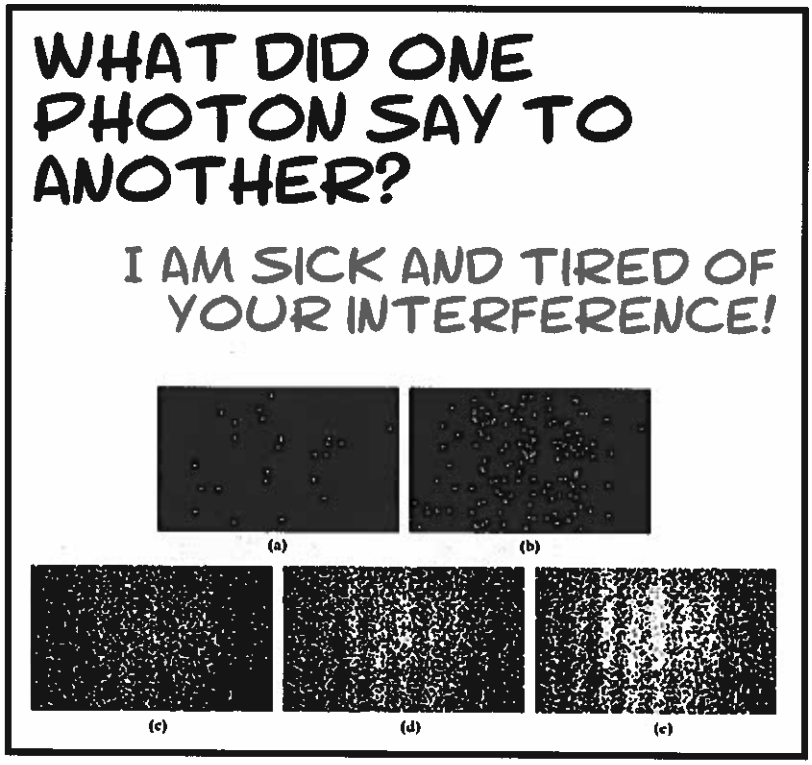
As the barrier length changes, the amount of phase picked up by the waves inside the two walls changes, leading to there constructive or destructive interference

Other evidence : two slit interference



Two slits creat two point sources with a stable relative phase. Then the electron density ~~deflected reflected~~ at the different points of the screen will depend on relative phase of two waves (travelling from the two sources/slits) to create constructive or destructive interference

Electron diffraction : rather than two slits, e-beam is scattered by a crystal, that works like a spatial diffraction grating, producing characteristic pattern that allows identification of the crystal structure of the target.



(a) – each particle detected individually as a dot

(e) – total probability function shows interference