WILLIAM & MARY

CHARTERED 1693

QUANTUM MECHANICS I NOTES

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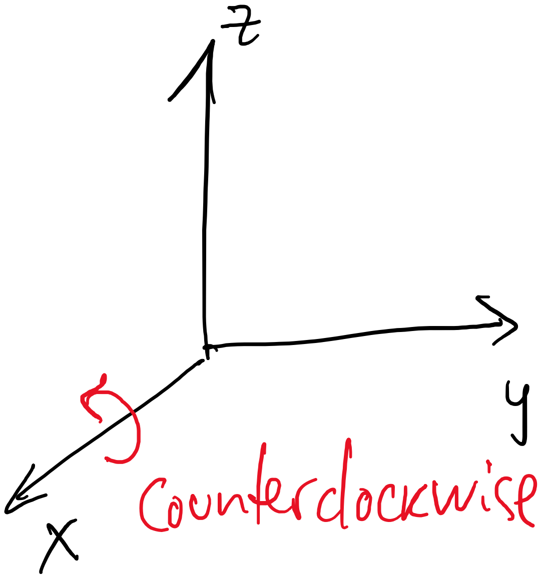
CHAPTER 2
MATRIX MECHANICS AND OPERATORS

Rotation Operators

$$\hat{R}(\varphi\,\hat{i}) = e^{-i\hat{J}_x\varphi/\hbar}$$



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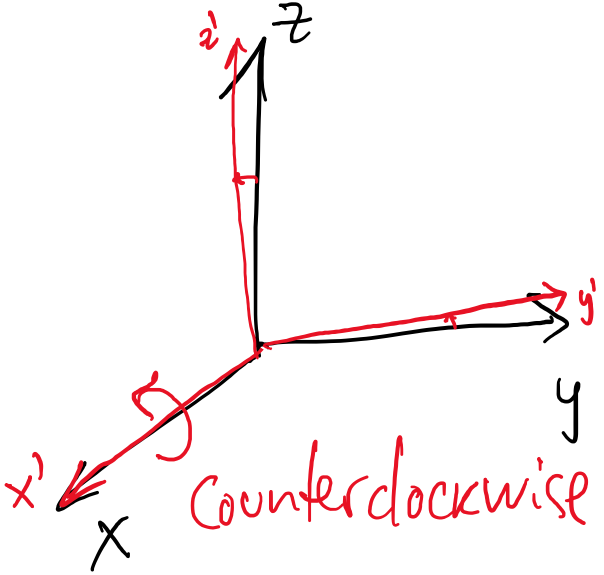

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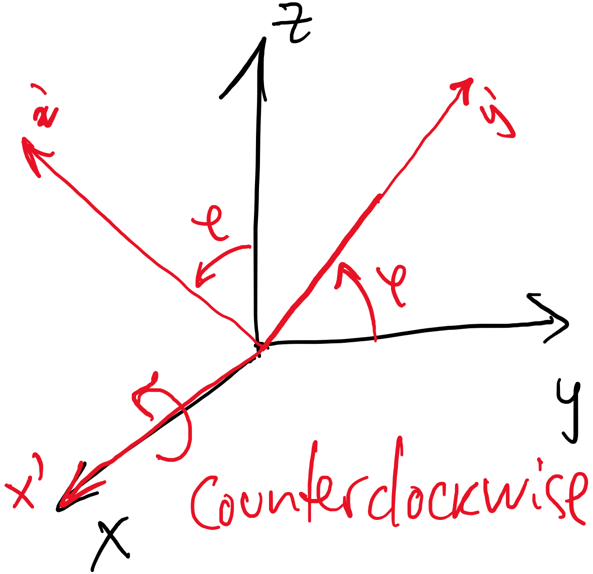
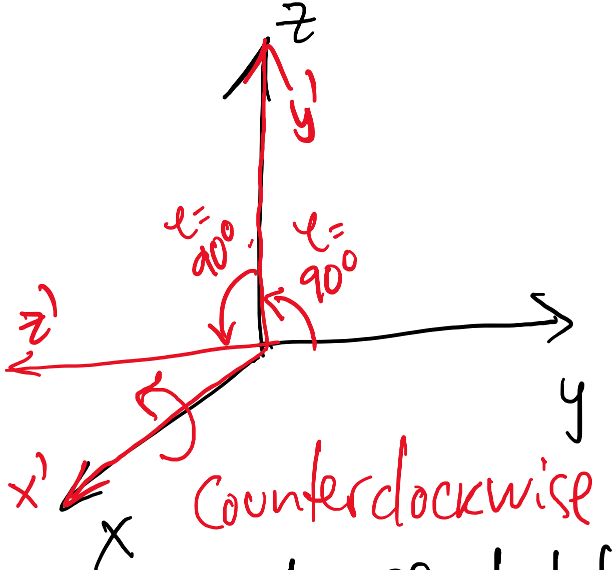


Rotation Operators

$$\hat{R}(\theta\,\hat{i}) = e^{-i\hat{J}_x\theta/\hbar}$$



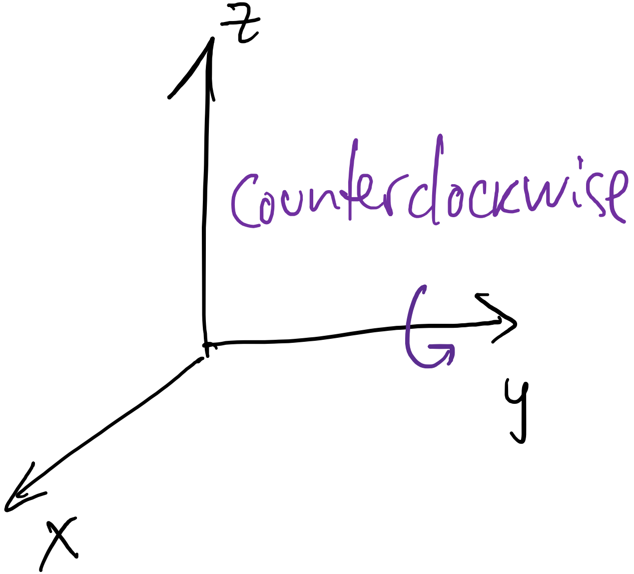
the axis x is not affected by the rotation about the x axis

x is an eigenvector of $\hat{R}(\theta\,\hat{i})$

Rotation Operators

$$\hat{R}(\theta\,\hat{i}) = e^{-i\hat{J}_x\theta/\hbar}$$

$$\hat{R}(\theta\,\hat{j}) = e^{-i\hat{J}_y\theta/\hbar}$$



Rotation Operators

$$\hat{R}(\varphi\,\hat{i}) = e^{-i\hat{J}_x \varphi/\hbar}$$

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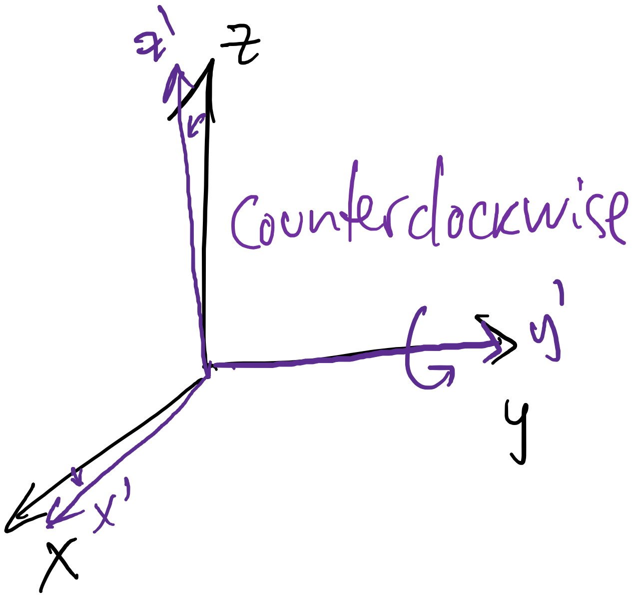

Rotation Operators

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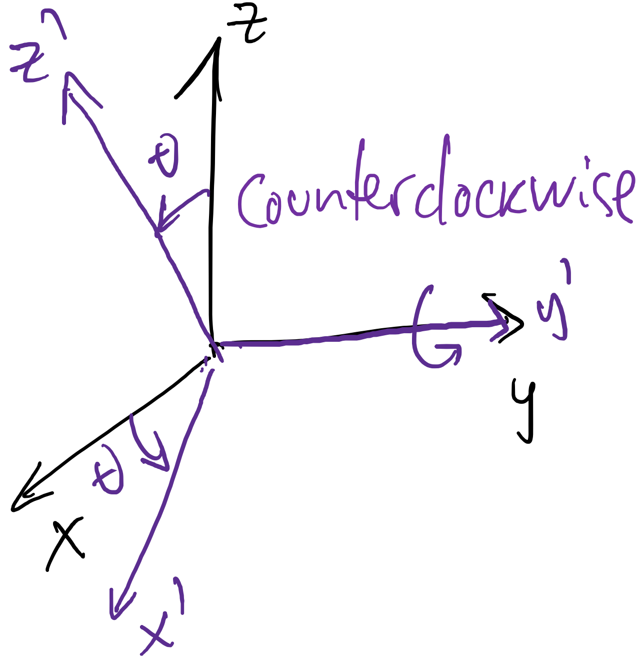

Rotation Operators

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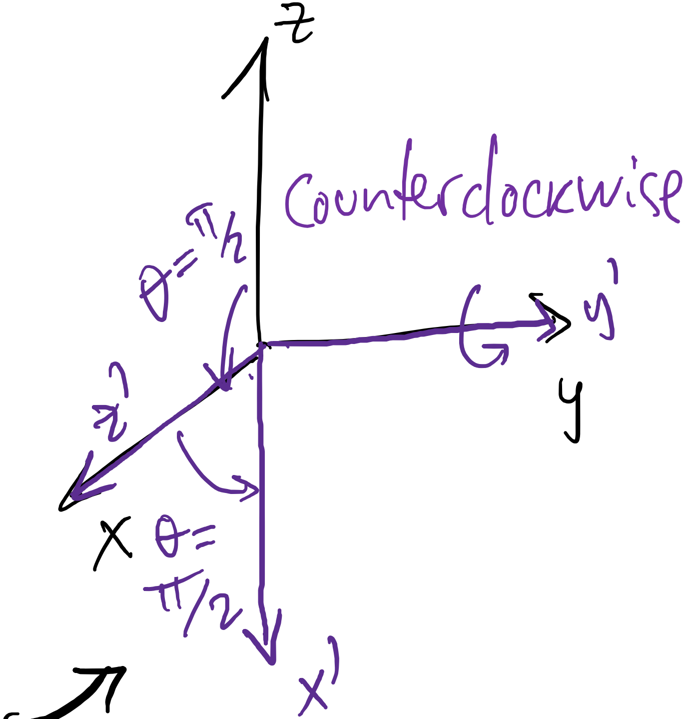


the axis y is not affected by the rotation about the y axis

y is an eigenvector of $\hat{R}(\theta\,\hat{j})$

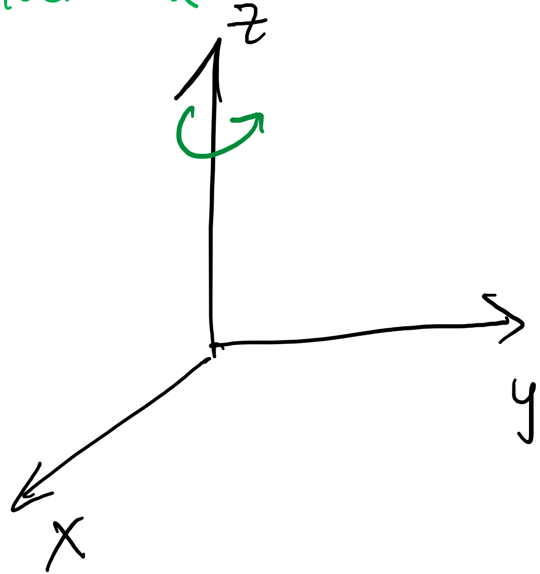
Rotation Operators

$$\hat{R}(\theta\,\hat{i}) = e^{-i\hat{J}_x\theta/\hbar}$$

$$\hat{R}(\theta\,\hat{j}) = e^{-i\hat{J}_y\theta/\hbar}$$

$$\hat{R}(\phi\,\hat{k}) = e^{-i\hat{J}_z\phi/\hbar}$$

Counterclockwise



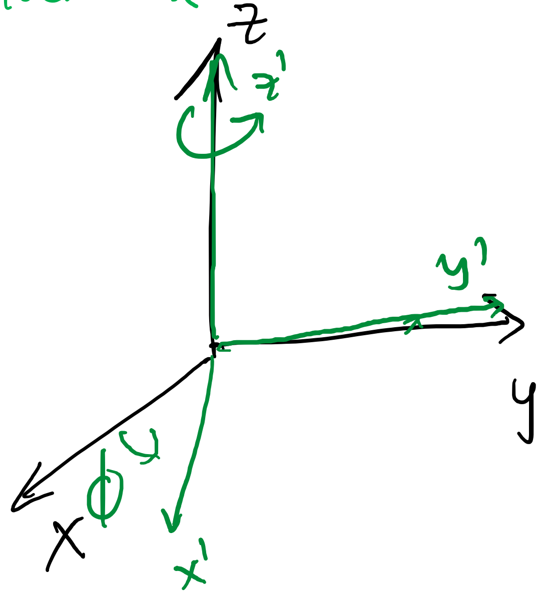
Rotation Operators

$$\hat{R}(\theta\,\hat{i}) = e^{-i\hat{J}_x \theta/\hbar}$$

$$\hat{R}(\theta\,\hat{j}) = e^{-i\hat{J}_y \theta/\hbar}$$

$$\hat{R}(\phi\,\hat{k}) = e^{-i\hat{J}_z \phi/\hbar}$$

Counterclockwise



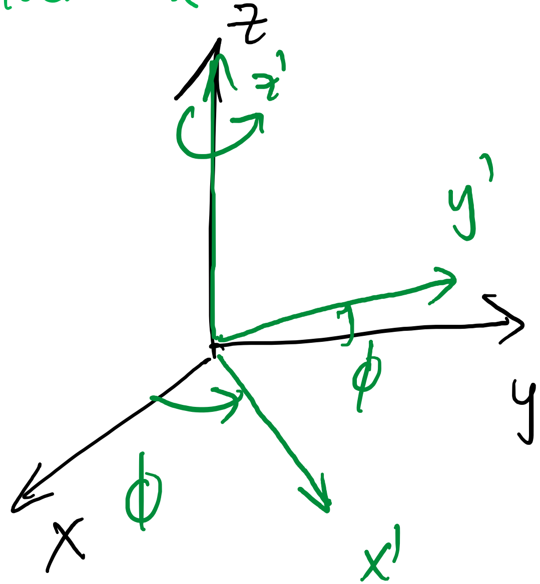
Rotation Operators

$$\hat{R}(\theta\,\hat{i}) = e^{-i\hat{J}_x\theta/\hbar}$$

$$\hat{R}(\theta\,\hat{j}) = e^{-i\hat{J}_y\theta/\hbar}$$

$$\vec{R}(\phi\,\vec{k}) = e^{-i\bar{J}_z\phi/\hbar}$$

Counterclockwise



Rotation Operators

$$\hat{R}(\theta\,\hat{i}) = e^{-i\hat{J}_x\theta/\hbar}$$

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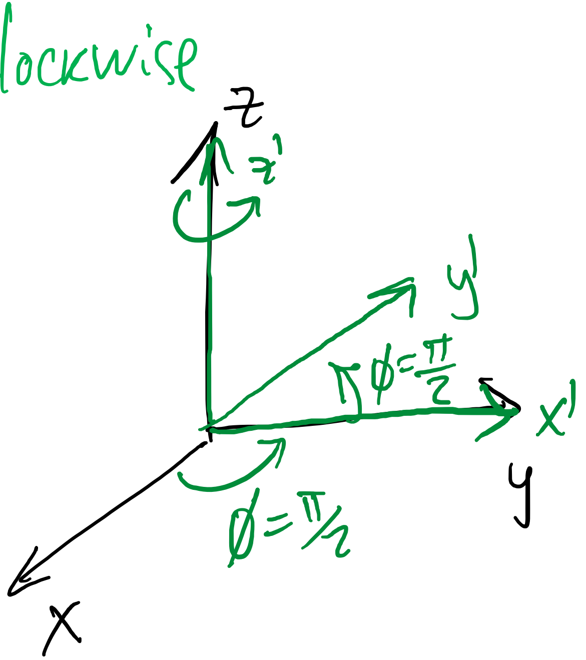

the axis z is not affected by the rotation about the z axis

z is an eigenvector of $\hat{R}(\phi\,\hat{k})$

Rotation in spin $\frac{1}{2}$ particles;

$$\hat{R}(\phi\hat{k})\,|+x\rangle = e^{-i\hat{J}_z\phi/\hbar}\,|+x\rangle = \left[1 - \frac{i\phi\hat{J}_z}{\hbar} + \frac{1}{2!}\left(-\frac{i\phi\hat{J}_z}{\hbar}\right)^2 + \cdots\right]|+x\rangle$$

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• Reminder: $|+x\rangle = \frac{1}{\sqrt{2}}|+z\rangle + \frac{1}{\sqrt{2}}|-z\rangle$

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$$\hat{R}(\phi\hat{k})\left[\frac{1}{\sqrt{2}}|+z\rangle + \frac{1}{\sqrt{2}}|-z\rangle\right] = \frac{1}{\sqrt{2}}\left[\underbrace{\hat{R}(\phi\hat{k})|+z\rangle}_{a} + \underbrace{\hat{R}(\phi\hat{k})|-z\rangle}_{b}\right]$$

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a $\hat{R}(\phi\hat{k})|+z\rangle = \left[1 - \frac{i\phi\hat{J}_z}{\hbar} + \frac{1}{2!}\left(-\frac{i\phi\hat{J}_z}{\hbar}\right)^2 + \cdots\right]|+z\rangle$

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(a) $$\hat{R}(\phi\hat{k})|+z\rangle = \left[1 - \frac{i\phi\hat{J}_z}{\hbar} + \frac{1}{2!}\left(-\frac{i\phi\hat{J}_z}{\hbar}\right)^2 + \cdots\right]|+z\rangle$$

$$= \left[|+z\rangle - \frac{i\phi}{\hbar}\hat{J}_z|+z\rangle + \frac{1}{2!}\left(-\frac{i\phi}{\hbar}\right)^2 \hat{J}_z^2|+z\rangle + \cdots\right]$$

Rotation in spin $\frac{1}{2}$ particles;

$$\hat{R}(\phi\hat{k})\,|+x\rangle = e^{-i\hat{J}_z\phi/\hbar}\,|+x\rangle = \left[1 - \frac{i\phi\hat{J}_z}{\hbar} + \frac{1}{2!}\left(-\frac{i\phi\hat{J}_z}{\hbar}\right)^2 + \cdots\right]|+x\rangle$$

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a $\quad \hat{R}(\phi\hat{k})|+z\rangle = \left[1 - \frac{i\phi\hat{J}_z}{\hbar} + \frac{1}{2!}\left(-\frac{i\phi\hat{J}_z}{\hbar}\right)^2 + \cdots\right]|+z\rangle$

$$= \left[|+z\rangle - \frac{i\phi}{\hbar}\hat{J}_z|+z\rangle + \frac{1}{2!}\left(-\frac{i\phi}{\hbar}\right)^2 \hat{J}_z^2|+z\rangle + \cdots\right]$$

$$= \left[|+z\rangle - \frac{i\phi}{\hbar}\left(\frac{\hbar}{2}\right)|+z\rangle + \frac{1}{2!}\left(-\frac{i\phi}{\hbar}\right)^2\left(\frac{\hbar}{2}\right)^2|+z\rangle + \cdots\right]$$

$$\hat{R}(\phi\hat{k})|+z\rangle = \left[|+z\rangle - \frac{i\phi}{\cancel{\hbar}}\left(\frac{\cancel{\hbar}}{2}\right)|+z\rangle + \frac{1}{2!}\left(-\frac{i\phi}{\cancel{\hbar}}\right)^2\left(\frac{\cancel{\hbar}}{2}\right)^2|+z\rangle + \cdots\right]$$

$$\hat{R}(\phi\hat{k})|+z\rangle = \left[|+z\rangle - \frac{i\phi}{\cancel{\hbar}}\left(\frac{\cancel{\hbar}}{2}\right)|+z\rangle + \frac{1}{2!}\left(-\frac{i\phi}{\cancel{\hbar}}\right)^2\left(\frac{\cancel{\hbar}}{2}\right)^2|+z\rangle + \cdots\right]$$

$$= \underbrace{\left[1 + \left(-\frac{i\phi}{2}\right) + \frac{1}{2!}\left(-\frac{i\phi}{2}\right)^2 + \cdots\right]}_{e^x = \sum_{N=0}^{\infty}\frac{x^N}{N!} \text{ with } x = -\frac{i\phi}{2}}|+z\rangle$$

a

$$\hat{R}(\phi\hat{k})|+z\rangle = e^{-i\phi/2}|+z\rangle$$

Next page ... $\Rightarrow$

$$\hat{R}(\phi\hat{k})|+z\rangle = \left[|+z\rangle - \frac{i\phi}{\cancel{\hbar}}\left(\frac{\cancel{\hbar}}{2}\right)|+z\rangle + \frac{1}{2!}\left(-\frac{i\phi}{\cancel{\hbar}}\right)^2\left(\frac{\cancel{\hbar}}{2}\right)^2|+z\rangle + \cdots\right]$$

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Next page ...⇒

$$\hat{R}(\phi\hat{k})|+z\rangle = \left[|+z\rangle - \frac{i\phi}{\cancel{\hbar}}\left(\frac{\cancel{\hbar}}{2}\right)|+z\rangle + \frac{1}{2!}\left(-\frac{i\phi}{\cancel{\hbar}}\right)^2\left(\frac{\cancel{\hbar}}{2}\right)^2|+z\rangle + \cdots\right]$$

$$= \underbrace{\left[1 + \left(-\frac{i\phi}{2}\right) + \frac{1}{2!}\left(-\frac{i\phi}{2}\right)^2 + \cdots\right]}_{e^x = \sum_{N=0}^{\infty} \frac{x^N}{N!} \text{ with } x = -\frac{i\phi}{2}}|+z\rangle$$

(a)

$$\hat{R}(\phi\hat{k})|+z\rangle = e^{-i\phi/2}|+z\rangle$$

$$\hat{R}(\phi\hat{k})\left[\frac{1}{\sqrt{2}}|+z\rangle + \frac{1}{\sqrt{2}}|-z\rangle\right] = \frac{1}{\sqrt{2}}\left[\underbrace{\hat{R}(\phi\hat{k})|+z\rangle}_{(a)} + \underbrace{\hat{R}(\phi\hat{k})|-z\rangle}_{(b)}\right]$$

(b) $$\hat{R}(\phi\hat{k})|-z\rangle = e^{i\phi/2}|-z\rangle$$

$$\hat{R}(\phi\hat{k})|+x\rangle = \frac{1}{\sqrt{2}}\left[e^{-i\phi/2}|+z\rangle + e^{i\phi/2}|-z\rangle\right]$$

Next page ... ⟹

$$\hat{R}(\phi\hat{k})|+x\rangle = \frac{1}{\sqrt{2}}\left[e^{-i\phi/2}|+z\rangle + e^{i\phi/2}|-z\rangle\right]$$

$$\hat{R}(\phi\hat{k})|+x\rangle = \frac{1}{\sqrt{2}}\left[e^{-i\phi/2}|+z\rangle + e^{i\phi/2}|-z\rangle\right]$$

$$= \frac{1}{\sqrt{2}}e^{-i\phi/2}\left[|+z\rangle + e^{i\phi}|-z\rangle\right]$$

$$\hat{R}(\phi\hat{k})|+x\rangle = \frac{e^{-i\phi/2}}{\sqrt{2}}\left[|+z\rangle + e^{i\phi}|-z\rangle\right] \; ;$$

$$\hat{R}(\phi\hat{k})|+x\rangle = \frac{1}{\sqrt{2}}\left[e^{-i\phi/2}|+z\rangle + e^{i\phi/2}|-z\rangle\right]$$

$$= \frac{1}{\sqrt{2}}e^{-i\phi/2}\left[|+z\rangle + e^{i\phi}|-z\rangle\right]$$

$$\hat{R}(\phi\hat{k})|+x\rangle = \frac{e^{-i\phi/2}}{\sqrt{2}}\left[|+z\rangle + e^{i\phi}|-z\rangle\right] \; ;$$

Rotation of $\phi = \pi/2$

$$\hat{R}(\phi\hat{k})|+x\rangle = \frac{1}{\sqrt{2}}\left[e^{-i\phi/2}|+z\rangle + e^{i\phi/2}|-z\rangle\right]$$

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$$\hat{R}(\phi\hat{k})|+x\rangle = \frac{e^{-i\phi/2}}{\sqrt{2}}\left[|+z\rangle + e^{i\phi}|-z\rangle\right] ;$$

Rotation of $\phi = \pi/2$

$$\hat{R}(\tfrac{\pi}{2}\hat{k})|+x\rangle = \frac{e^{-i\pi/4}}{\sqrt{2}}\left[|+z\rangle + e^{i\pi/2}|-z\rangle\right]$$

$$\hat{R}(\phi\hat{k})|+x\rangle = \frac{1}{\sqrt{2}}\left[e^{-i\phi/2}|+z\rangle + e^{i\phi/2}|-z\rangle\right]$$

$$= \frac{1}{\sqrt{2}}e^{-i\phi/2}\left[|+z\rangle + e^{i\phi}|-z\rangle\right]$$

$$\hat{R}(\phi\hat{k})|+x\rangle = \frac{e^{-i\phi/2}}{\sqrt{2}}\left[|+z\rangle + e^{i\phi}|-z\rangle\right] ;$$

Rotation of $\phi = \pi/2$

$$\hat{R}(\tfrac{\pi}{2}\hat{k})|+x\rangle = \frac{e^{-i\pi/4}}{\sqrt{2}}\left[|+z\rangle + e^{i\pi/2}|-z\rangle\right]$$

Reminder:

$$\boxed{e^{i\frac{\pi}{2}} = \cancelto{0}{\cos\frac{\pi}{2}} + i\,\cancelto{1}{\sin\frac{\pi}{2}}}$$

$$\hat{R}(\phi\hat{k})|+x\rangle = \frac{1}{\sqrt{2}}\left[e^{-i\phi/2}|+z\rangle + e^{i\phi/2}|-z\rangle\right]$$

$$= \frac{1}{\sqrt{2}}e^{-i\phi/2}\left[|+z\rangle + e^{i\phi}|-z\rangle\right]$$

$$\hat{R}(\phi\hat{k})|+x\rangle = \frac{e^{-i\phi/2}}{\sqrt{2}}\left[|+z\rangle + e^{i\phi}|-z\rangle\right];$$

Rotation of $\phi = \pi/2$

$$\hat{R}(\tfrac{\pi}{2}\hat{k})|+x\rangle = \frac{e^{-i\pi/4}}{\sqrt{2}}\left[|+z\rangle + e^{i\pi/2}|-z\rangle\right]$$

Reminder:

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$$\hat{R}(\tfrac{\pi}{2}\hat{k})|+x\rangle = \frac{e^{-i\pi/4}}{\sqrt{2}}\underbrace{\left[|+z\rangle + i|-z\rangle\right]}_{|+y\rangle \text{ in the z basis}}$$

$$\hat{R}(\phi\hat{k})|+x\rangle = \frac{1}{\sqrt{2}}\left[e^{-i\phi/2}|+z\rangle + e^{i\phi/2}|-z\rangle\right]$$

$$= \frac{1}{\sqrt{2}}e^{-i\phi/2}\left[|+z\rangle + e^{i\phi}|-z\rangle\right]$$

$$\hat{R}(\phi\hat{k})|+x\rangle = \frac{e^{-i\phi/2}}{\sqrt{2}}\left[|+z\rangle + e^{i\phi}|-z\rangle\right];$$

Rotation of $\phi = \pi/2$

$$\hat{R}(\tfrac{\pi}{2}\hat{k})|+x\rangle = \frac{e^{-i\pi/4}}{\sqrt{2}}\left[|+z\rangle + e^{i\pi/2}|-z\rangle\right]$$

Reminder:

$$\boxed{e^{i\frac{\pi}{2}} = \cos\frac{\pi}{2} + i\sin\frac{\pi}{2}}$$

$$\hat{R}(\tfrac{\pi}{2}\hat{k})|+x\rangle = \frac{e^{-i\pi/4}}{\sqrt{2}}\underbrace{\left[|+z\rangle + i|-z\rangle\right]}_{|+y\rangle \text{ in the } z \text{ basis}} = e^{-i\pi/4}|+y\rangle$$

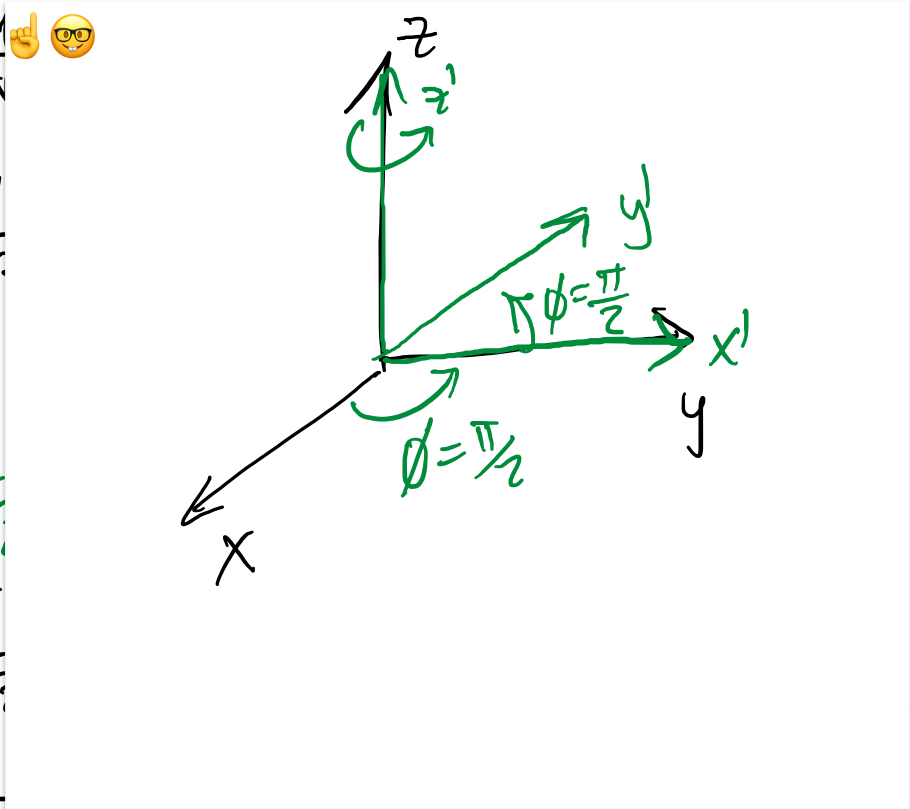
$\hat{R}(\phi\hat{k})|+x\rangle = \dots$

$= \dots$

$\hat{R}(\phi\hat{k})|+x\rangle$

Rotation of $\phi = \pi\dots$

$\hat{R}(\frac{\pi}{2}\hat{k})|+x\rangle = \frac{e^{-i\dots}}{\sqrt{\dots}}$



$\cancelto{0}{\cos\frac{\pi}{2}} + i\cancelto{1}{\sin\frac{\pi}{2}}$

$\hat{R}(\frac{\pi}{2}\hat{k})|+x\rangle = \frac{1}{\sqrt{2}}\underbrace{[\dots]}_{|+y\rangle \text{ in the z basis}} = e^{\dots}|+y\rangle$

$\hat{R}(\phi\hat{k})|+x\rangle = \frac{1}{\sqrt{}}$

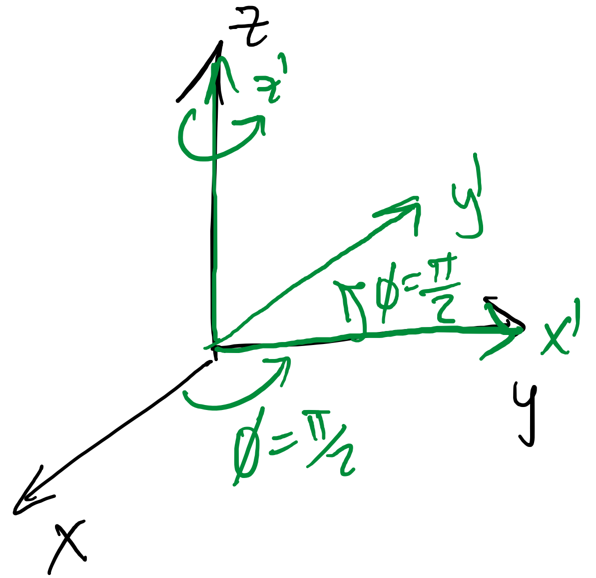
$= \frac{1}{\sqrt{}}$

$\hat{R}(\phi\hat{k})|+x\rangle$

Rotation of $\phi = \pi$

$\hat{R}(\frac{\pi}{2}\hat{k})|+x\rangle = \frac{e^{-i}}{\sqrt{}}$

$\hat{R}(\frac{\pi}{2}\hat{k})|+x\rangle = \frac{1}{\sqrt{2}}\underbrace{[\ldots]}_{|+y\rangle \text{ in the z basis}} = e^{\ldots}|+y\rangle$



a

$\hat{R}(\phi\hat{k})|+z\rangle = e^{-i\phi/2}|+z\rangle$ if $\phi = 2\pi$

$\cos\cancel{\frac{\pi}{2}}^{\,0} + i\sin\cancel{\frac{\pi}{2}}^{\,1}$

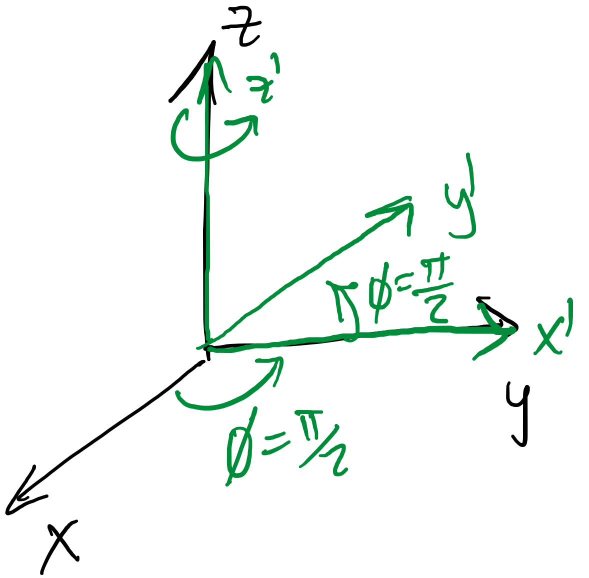
$\hat{R}(\phi\hat{k})|+x\rangle = \frac{1}{\sqrt{\ }}\ldots$

$= \frac{1}{\sqrt{\ }}\ldots$

$\hat{R}(\phi\hat{k})|+x\rangle$

Rotation of $\phi = \frac{\pi}{\ }\ldots$

$\hat{R}(\frac{\pi}{2}\hat{k})|+x\rangle = \frac{e^{-i\ldots}}{\sqrt{\ }}\ldots$



a

$\hat{R}(\phi\hat{k})|+z\rangle = e^{-i\phi/2}|+z\rangle$ if $\phi = 2\pi$

$R(2\pi\hat{k})|+z\rangle = -|+z\rangle$

$\cancelto{0}{\cos\frac{\pi}{2}} + i\cancelto{1}{\sin\frac{\pi}{2}}$

$\hat{R}(\frac{\pi}{2}\hat{k})|+x\rangle = \frac{1}{\sqrt{2}}\underbrace{\left[\ldots\right]}_{|+y\rangle \text{ in the z basis}} = e^{\ldots}|+y\rangle$

In a Two-dimensional basis:

Identity operator $\rightarrow$ $\underset{\text{ket}}{|+z\rangle}\underset{\text{bra}}{\langle +z|} + \underset{\text{ket}}{|-z\rangle}\underset{\text{bra}}{\langle -z|}$

In a Two-dimensional basis:

Identity operator $\rightarrow$ $|{+z}\rangle\langle{+z}| + |{-z}\rangle\langle{-z}|$

ket bra ket bra

what is the effect of this operator on a Quantum state:

In a Two-dimensional basis

Identity operator $\rightarrow$ $\underbrace{|+z\rangle}_{\text{ket}}\underbrace{\langle +z|}_{\text{bra}} + \underbrace{|-z\rangle}_{\text{ket}}\underbrace{\langle -z|}_{\text{bra}}$

what is the effect of this operator on a Quantum state:

$$\left[\,|+z\rangle\langle +z| + |-z\rangle\langle -z|\,\right]\left[\,c_+|+z\rangle + c_-|-z\rangle\,\right] =$$

In a Two-dimensional basis:

Identity operator $\rightarrow$ $\underset{\text{ket}}{|+z\rangle}\underset{\text{bra}}{\langle +z|} + \underset{\text{ket}}{|-z\rangle}\underset{\text{bra}}{\langle -z|}$

what is the effect of this operator on a Quantum state:

$$\left[|+z\rangle\langle +z| + |-z\rangle\langle -z|\right]\left[C_+|+z\rangle + C_-|-z\rangle\right] =$$

$$= C_+|+z\rangle + C_-|-z\rangle$$

In a Two-dimensional basis

Identity operator $\rightarrow$ $|+z\rangle\langle+z| + |-z\rangle\langle-z|$

ket bra ket bra

what is the effect of this operator on a Quantum state:

$$\left[|+z\rangle\langle+z| + |-z\rangle\langle-z|\right]\left[C_+|+z\rangle + C_-|-z\rangle\right] =$$

$$= C_+|+z\rangle + C_-|-z\rangle$$

It did nothing to the Quantum state $|\psi\rangle$

In a Two-dimensional basis:

Identity operator $\rightarrow$ $|+z\rangle\langle +z| + |-z\rangle\langle -z|$

ket bra ket bra

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ket bra ket bra

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$$\hat{P}_+ + \hat{P}_- = 1$$

$$|{+z}\rangle\langle{+z}| + |{-z}\rangle\langle{-z}| = 1$$

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$$\hat{P}_+ |\psi\rangle = |+z\rangle\langle +z| \left[c_+ |+z\rangle + c_- |-z\rangle \right]$$

$$= c_+ |+z\rangle$$ projects out the component of $|\psi\rangle$ along the $|+z\rangle$ direction

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Completeness relation

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we can also express the identity operator as

$$|+x\rangle\langle +x| + |-x\rangle\langle -x| = 1 \quad \text{or} \quad |+y\rangle\langle +y| + |-y\rangle\langle -y| = 1$$

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Propertys:

$\hat{P}_+^2 = \hat{P}_+$ and $\hat{P}_-^2 = \hat{P}_-$ also $\hat{P}_+\hat{P}_- = 0$ and $\hat{P}_-\hat{P}_+ = 0$

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In general: operators do not commute

Operator on bras $\langle \psi|$

$\tilde{A}|\psi\rangle \longrightarrow \langle\psi|\tilde{A}^\dagger$ in the case of $\hat{P}_\pm \longrightarrow \langle\psi|\hat{P}_\pm$