WILLIAM & MARY

CHARTERED 1693

QUANTUM MECHANICS I NOTES

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SUMMARY OF CH1

Quantum States:

Spin States: z direction $|\psi\rangle = C_+|+z\rangle + C_-|-z\rangle$

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