



WILLIAM & MARY

CHARTERED 1693

QUANTUM MECHANICS I NOTES

09/13/2023

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SUMMARY OF CH1

Quantum States:

Spin States: z direction

$$|\psi\rangle = c_+|+\rangle + c_-|-\rangle$$

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$|+z\rangle, |-z\rangle \rightarrow$ Orthogonal basis

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all the elements of the basis

in the same way

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We saw that the elements of the basis are normalized and orthogonal; meaning that

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$$\langle \psi | = \sum_n \langle \psi | a_n \rangle \langle a_n | \quad ; \text{ also } \langle \psi | a_n \rangle = \langle a_n | \psi \rangle^*$$

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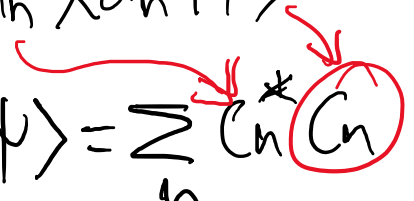
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