WILLIAM & MARY

CHARTERED 1693

QUANTUM STATES AS VECTORS

09/11/2023

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Expectation Value and uncertainty

$$|\chi\rangle = \frac{1}{2}|+z\rangle + i\,\frac{\sqrt{3}}{2}|-z\rangle$$

- what is the spin value (along $\hat{z}$) of the particle?
- Can we know it by performing one measurement?

Best I can do is the Average

QUICK REVIEW ABOUT MEASUREMENTS…

Probability

In a set of N measurements we can calculate the standar deviation as:

$$\sigma = \sqrt{\langle j^2 \rangle - \langle j \rangle^2}$$

Average of the squares

Average square

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- $$\langle j^2 \rangle = \sum_j \frac{N_j}{N} j^2$$

all the results

of all those measurements, how many times I measured the value "j"

total number of measurements

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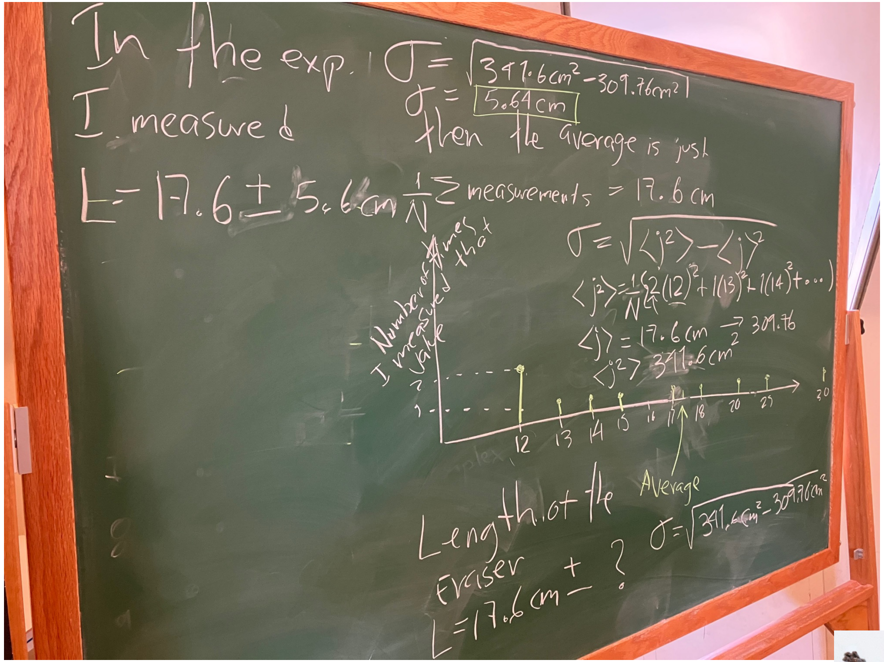
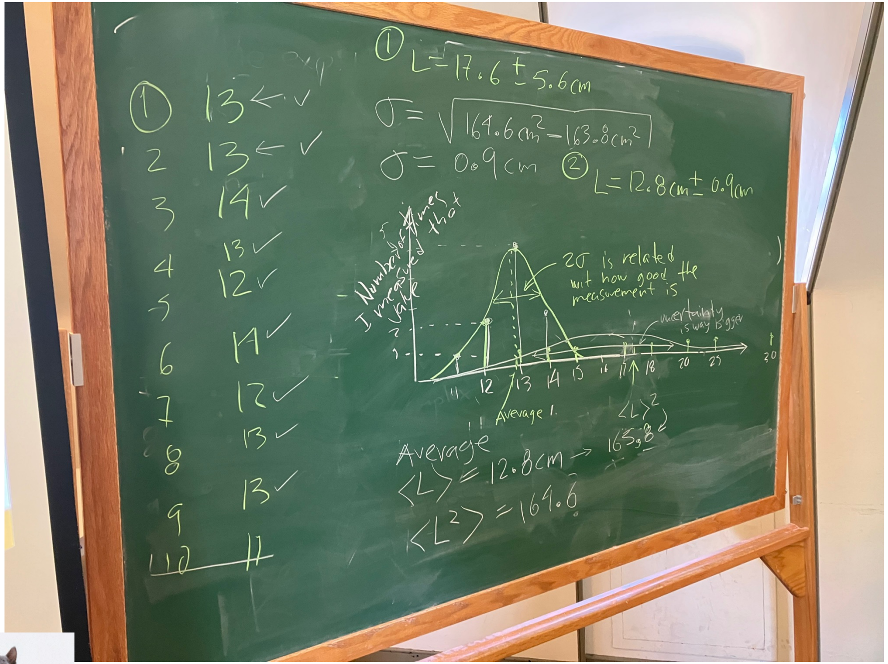
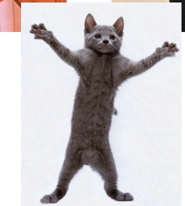
Lets calculate the standard deviation of the example experiment 🙂

EXPERIMENT 1…



EXPERIMENT 2…





QUICK REVIEW ABOUT MEASUREMENTS… THE END

• what is the spin value (along $\hat{z}$) of the particle?

$$|\psi\rangle = \frac{1}{2}|+z\rangle + i\frac{\sqrt{3}}{2}|-z\rangle$$

We do calculate the expectation value

$$\langle S_z\rangle = |\langle +z|\psi\rangle|^2 \left(\frac{\hbar}{2}\right) + |\langle -z|\psi\rangle|^2\left(-\frac{\hbar}{2}\right)$$

$$\langle S_z\rangle = \frac{1}{4}\left(\frac{\hbar}{2}\right) - \frac{3}{4}\left(\frac{\hbar}{2}\right)$$

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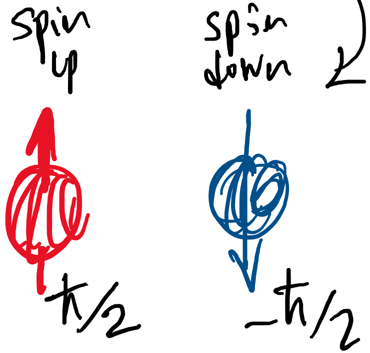

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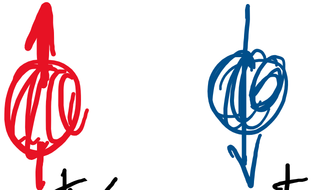

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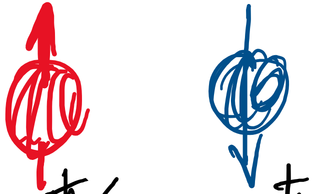

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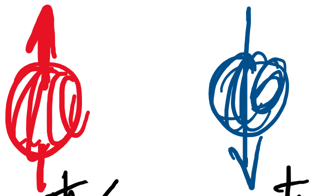

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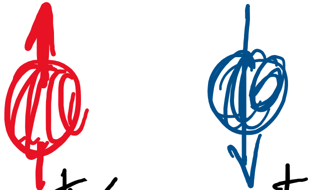

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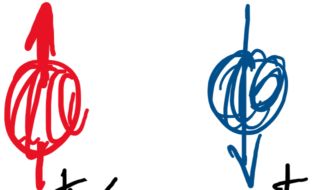

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$$\Delta S_z = 0.43\hbar$$

we found that the particles have a spectation value of $-\frac{\hbar}{4}$ for the spin in the z direct. and an uncertainty of $0.43\hbar$

particle in a state $|\psi\rangle = \frac{1}{2}|+z\rangle + i\frac{\sqrt{3}}{2}|-z\rangle$; what is the probability of finding $\frac{\hbar}{2}$ after a S_y measurement

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in that way $\langle +y|\psi\rangle = \frac{1}{2}$ and $|\langle +y|\psi\rangle|^2 = \frac{1}{4}$; meaning that the probability of finding $\frac{\hbar}{2}$ is 25%

😎 Piece of cake!

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$\frac{\hbar}{2}$ Corres... ...late

$|\langle +$ ... $\frac{\sqrt{3}}{2}\langle +y|-z\rangle$

We are pe...

a S_y m...

Actually:

- $\langle +y|+z\rangle \neq 1$ it is not equal to 1 because of the normalization
- $\langle +y|-z\rangle \neq 0$ they are in general not orthogonal

in that way $\langle +y|\psi\rangle = \frac{1}{2}$ and $|\langle +\psi|\psi\rangle| = \frac{1}{4}$;

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Wrong

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$$\langle +y|\psi\rangle = \frac{1}{\sqrt{2}}\cdot\frac{1}{2}\underbrace{\langle +z|+z\rangle}_{1} + \frac{1}{\sqrt{2}}\cdot\frac{\sqrt{3}}{2}\underbrace{\langle +z|-z\rangle}_{=0} - \frac{i}{\sqrt{2}}\cdot\frac{1}{2}\underbrace{\langle -z|+z\rangle}_{=0} - \frac{i}{\sqrt{2}}\frac{i\sqrt{3}}{2}\underbrace{\langle -z|-z\rangle}_{1}$$

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$$- \frac{i}{\sqrt{2}}\cdot\frac{1}{2}\cancel{\langle -z|+z\rangle}^{=0} - \frac{i}{\sqrt{2}}\frac{i\sqrt{3}}{2}\cancel{\langle -z|-z\rangle}^{1}$$

then

$$\langle +y|\psi\rangle = \left(\frac{1}{\sqrt{2}}\right)\left(\frac{1}{2}\right) - \left(\frac{i}{\sqrt{2}}\right)\left(\frac{i\sqrt{3}}{2}\right)$$

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$$|\langle +y|\psi\rangle|^2 = \frac{1}{2} + \frac{\sqrt{3}}{4} = 0.93$$

$|\langle +y|\psi\rangle|^2 = 0.93$; probability is 93% !!

What is the probability of $-\frac{\hbar}{2}$ (spin down) of a S_y measurement?

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$$\left(\frac{1}{2} - \frac{\sqrt{3}}{4}\right)\left(-\frac{\hbar}{2}\right) + \left(\frac{1}{2} + \frac{\sqrt{3}}{4}\right)\left(\frac{\hbar}{4}\right)$$

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Is it possible to reduce the uncertainty in QM? Is there any effect?

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Spoiler Alert!

SUMMARY OF CH1