QM I review

Basic quantum system

- Quantized spins: spin-1/2 particle
  (spin angular momentum) spin-1 particle
  $\hat{S}^2, \hat{S}_x, \hat{S}_y, \hat{S}_z, \hat{S}_+, \hat{S}_-$
- ~~Angular~~ Orbital angular momentum
  $\hat{L}^2, \hat{L}_x, \hat{L}_y, \hat{L}_z$

- 1D particle motion $\hat{x}, \hat{p}_x$
- 3D particle motion $\hat{x}, \hat{y}, \hat{z}$ or $\hat{r}, \hat{\theta}, \hat{\varphi}$
  $\hat{p}_x, \hat{p}_y, \hat{p}_z$ $\hat{p}^2, \hat{L}^2, \hat{L}_z$

Main concepts

Quantum state $\longleftrightarrow$ measurements (the only way to extract information)

we use a basis of some ~~some~~ measurement to ~~characterize~~ label the state

Measurements and actions are represented as operators $\hat{A}$ ($\hat{x}, \hat{p}_x, \hat{H}, \hat{L}^2, \hat{S}_x, \hat{S}_z, \hat{S}^2 \ldots$)

Typically, we choose as the basis the eigenstates of a particular operator

If the basis is the eigen-basis for the operator

$\hat{A}|\psi_n\rangle = A_n|\psi_n\rangle$

Then (and only then)

$\langle\hat{A}\rangle = \sum_n A_n P_n = \sum_n A_n |C_n|^2$

$\cancel{\hat{A}\psi(x) = \alpha\psi(x)}$ i.e. $\hat{V}(\vec{x}) = \frac{1}{2}m\omega^2\vec{x}^2$

$\hat{A}\psi(x) = \alpha(x)\cdot\psi(x)$

$\langle\hat{A}\rangle = \int_{-\infty}^{\infty} \alpha(x)|\psi(x)|^2 dx$

Time evolution

General description (always valid)

$$i\hbar\frac{\partial\psi(x,t)}{\partial t} = \hat{H}\psi(x,t)$$

$$\psi(x,t) = \underbrace{e^{-i\hat{H}t/\hbar}}_{\text{operator exponent!}}\psi(x,t=0)$$

Most common approach: find stationary states (eigenstates of a Hamiltonian)

$$\hat{H}\psi_E(x) = E\psi_E(x) \quad \text{or} \quad \hat{H}|E_n\rangle = E_n|E_n\rangle$$

$$\psi_E(x,t) = \underbrace{e^{-iEt/\hbar}}_{\text{algebraic exponent}}\psi_E(x) \qquad |E_n\rangle(t) = e^{-iE_nt/\hbar}|E_n\rangle$$

$|\psi_E(x,t)|^2 = |\psi_E(x)|^2 \rightarrow$ probability distribution does not change

"Short cut" to figure out a state evolution

$|\psi\rangle = \sum_n c_n|\psi_n\rangle$

if $\{|\psi_n\rangle\}$ are eigenstates of $\hat{H}|\psi_n\rangle = E_n|\psi_n\rangle$

$$|\psi(t)\rangle = \sum_n c_n|\psi_n(t)\rangle = \sum_n c_n e^{-iE_nt/\hbar}|\psi_n\rangle$$

Our most popular measurements

Spin measurements: $\boxed{S_z \text{ basis}}$ $\hat{S}_z|\pm z\rangle = \pm\frac{\hbar}{2}|\pm z\rangle$

S_x basis $\hat{S}_x|\pm x\rangle = \pm\frac{\hbar}{2}|\pm x\rangle$

S_y basis

Pick the basis $\rightarrow$ Write the state ~~of~~ $\rightarrow$
$\hat{S}_z$ in this basis
$|\pm z\rangle$

$|\pm x\rangle = \frac{1}{\sqrt{2}}(|+z\rangle \pm |-z\rangle)$

~~$|\psi\rangle = c_1|+z\rangle + c_2|-z\rangle$~~

$\rightarrow$ Use any operator representation in this basis

$$\hat{S}_y = \frac{\hbar}{2}\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \leftarrow \text{in z-basis}$$

In general

discrete basis $\{|\psi_n\rangle\}$

$$|\psi\rangle = \sum_{n=1}^{N} c_n |\psi_n\rangle = \begin{pmatrix} c_1 \\ c_2 \\ \vdots \\ c_N \end{pmatrix}$$

$c_n = \langle\psi_n|\psi\rangle$ ↙ complex number

Probability to be at $|\psi_n\rangle$

$P_n = |\langle\psi_n|\psi\rangle|^2$ ↙ non-negative real number

Operator matrix elements

$$A_{mn} = \langle\psi_m|\hat{A}|\psi_n\rangle$$

$$\hat{A} = \sum_{m,n=1}^{N} A_{mn}|\psi_m\rangle\langle\psi_n|$$

continuous basis $|x\rangle$

$\psi(x) = \langle x|\psi\rangle = f(x)$

~~$\psi(x)$ - probability density~~

Probability to be at ~~$a<x<b$~~

$$P_{[a,b]} = \int_a^b |\psi(x)|^2 dx$$

Operators $\rightarrow$ actions of ~~functions~~

$$\hat{p}_x = -\frac{i}{\hbar}\frac{\partial}{\partial x}$$

Operator average value in state $|\psi\rangle$

$$\langle\hat{A}\rangle = \langle\psi|\hat{A}|\psi\rangle$$

$$\langle\hat{A}\rangle = \int_{-\infty}^{+\infty}\psi^*(x)\hat{A}\psi(x)\,dx$$

"Main" Hamiltonians

1D Constant potential (step, well, bump)

classically allowed region & positive kinetic energy
oscillating solutions
sin & cos or e^{ikx}, e^{-ikx}

classically forbidden region ($K<0$)
decaying or growing exponents

at the step → boundary conditions
$\psi(x)$ continuous (always)
$\psi'(x)$ continuous (unless $U(x)=\infty$)

1D Harmonic oscillator
$E_n = \hbar\omega(n+\frac{1}{2})$
$\hat{a}|n\rangle = \sqrt{n}\,|n-1\rangle$
$\hat{a}^+|n\rangle = \sqrt{n+1}\,|n+1\rangle$

3D Rectangular wells (or 2D) → independent motion $\psi_{n_x,n_y,n_z}(x,y,z) = \psi_{n_x}(x)\,\psi_{n_y}(y)\,\psi_{n_z}(z)$

3D Coulumb potential $|n,l,m\rangle$
$\hat{H} = -ke^2/r$
$H|n,l,m\rangle = E_n|n,l,m\rangle$
$E_n = -\frac{E_R}{n^2}$ $l=0,..\,n-1$
$m=0,\pm1,..\pm l$

Spin or orbital angular momentum in the magnetic field $\hat{H} = -\vec{\mu}\,\vec{B} = -\mu_B g(\vec{L}+2\vec{S})\cdot\vec{B}$

$\hat{H}|n,l,m\rangle = -\mu_B g\, L_z\cdot B_z = -\mu_B g\hbar B_z\cdot \underline{m}\,|n,l,m\rangle$

$\hat{H}\,|S_z=\pm\frac{1}{2}\rangle = -2\mu_B g\, S_z\,|S_z=\pm1/2\rangle$

Rotational motion $\hat{H} = \frac{L^2}{2I}$
$\hat{H}|n,l,m\rangle = \frac{\hbar^2 l(l+1)}{2I}$

Topics proposed by students as possible questions on the final

Proposed answers	**Likely to appear?**
Stern-Gerlach apparatus	yes
Uncertainty principle	maybe
Eigenvector and eigenvalues calculations	yes
Quantum tunneling	no
Perturbation theory	no
3D square well	maybe
Hydrogen atom	yes
Spectroscopy	yes
Angular momentum	yes
Time evolution	yes
SHO	yes
Transmission/reflection coefficients	yes
Two-particle system	maybe
Gaussian wave packet	maybe
Spherical well	no
Expectation values	yes
Commutation relations	maybe
Spin ½, Spin 1 particles	yes
Spins in magnetic fields	yes
Diatomic systems	maybe
1D square well	yes
Entanglement	maybe

Professor Novikova's top list of things to appear in the final (only some will be included, though)

Quantum systems	**Types of calculations**
Spin ½, Spin 1 particles	State decomposition, basis change
Spins in magnetic fields	Eigenvector and eigenvalues calculations
1D, 2D, 3D infinite square well	Expectation values
Scattering in a square step/barrier	Commutation relations
SHO	Time evolution
Two-particle system	Uncertainty principle
Angular momentum	Spectroscopy
Hydrogen atom	
Diatomic systems (rotators, oscillators)	
Entangled spins	