PHYS 313: Quantum Mechanics I

Problem set # 7 **(updated)** *(due November 15)*

All problems are mandatory, unless marked otherwise. Each problem is 10 points.

Townsend, Ch. 7: 7.7, 7.9, 7.13, 7.14

Q1 A particle of mass m is trapped in the harmonic potential, a at time $t = 0$ it is found in a state
$\psi(x) = C\big(1 + \sqrt{\frac{4m\omega}{\hbar}}x\big)e^{-\frac{m\omega}{2\hbar}x^2}$.
(a) Find the normalization constant C.
(b) If the energy of this particle is measured, what are the possible outcomes and what are their probabilities?
(c) At a later time T the wave function is $\psi(x,T) = C_1\big(1+i\sqrt{\frac{4m\omega}{\hbar}}x\big)e^{-\frac{m\omega}{2\hbar}x^2}$. What is the smallest possible value of T? What is C_1/C?
Hint: You may find useful to recall the first few known eigenstates of an oscillators:
$\psi_0(x) = \sqrt[4]{\frac{m\omega}{\pi\hbar}}e^{-\frac{m\omega}{2\hbar}x^2},\ E_0 = \frac{1}{2}\hbar\omega$
$\psi_1(x) = \sqrt[4]{\frac{m\omega}{\pi\hbar}}\sqrt{\frac{2m\omega}{\hbar}}xe^{-\frac{m\omega}{2\hbar}x^2},\ E_1 = \frac{3}{2}\hbar\omega$
$\psi_2(x) = \sqrt[4]{\frac{m\omega}{4\pi\hbar}}\big(\frac{2m\omega}{\hbar}x^2 - 1\big)e^{-\frac{m\omega}{2\hbar}x^2},\ E_2 = \frac{5}{2}\hbar\omega$

Q2 Not so simple harmonic oscillator. Imagine a one-sided oscillator (like a spring that can stretch but not compress):
$$V(x) = \begin{cases} m\omega^2x^2/2 & x \geq 0 \\ +\infty & x < 0 \end{cases}$$
What are the energies and the wavefunctions corresponding to stationary states of a particle in such a potential? Note, that this problem is about reasoning, you don't need to do actual calculations.