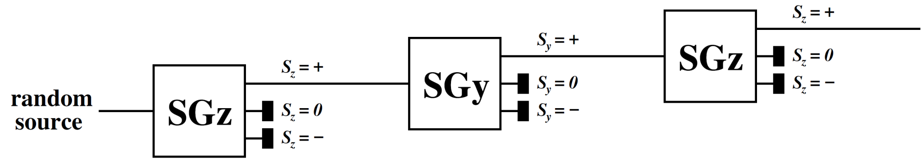
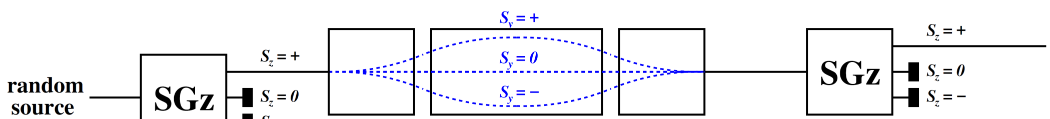
**Problem Q1**

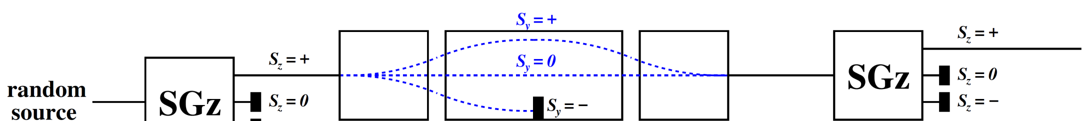



(a) Since the original particle spin is oriented randomly, there is $1/3$ probability of its passing through the first SG apparatus. After that the probability of its passing through the second apparatus $P_2 = |\langle y_{+1}|1,1\rangle|^2 = 1/4$, and the probability of it making through the third SG apparatus is $P_3 = |\langle 1,1|y_{+1}\rangle|^2 = 1/4$. Thus, the total detection probability is $P_{tot} = P_1 \times P_2 \times P_3 = 1/48$.




**Modified SGy**

(b) The modified SGy apparatus separates and then recombines all the spin states without modification, so at its output the state is unchanged:
$|y_{+1}\rangle\langle y_{+1}|1,1\rangle + |y_0\rangle\langle y_0|1,1\rangle + |y_{-1}\rangle\langle y_{-1}|1,1\rangle = \hat{1}|1,1\rangle$.
Thus all of the particles transmitted through the first apparatus will make it through the system, and the total detection probability is $P_{tot} = 1/3$.




**modified Modified SGy**

(c) Now one of the passes inside the modified SGy is blocked, but we don't know which of two paths inside the particle took. So we have to properly add the wavefunctions. Thus, at the output of the second apparatus the particle state is:

$$|out\rangle = |y_{+1}\rangle\langle y_{+1}|1,1\rangle + |y_0\rangle\langle y_0|1,1\rangle = \begin{pmatrix} 3/4 \\ i/2\sqrt{2} \\ 1/4 \end{pmatrix}.$$

Using this, we can find that the probability of passing through the third apparatus is: $P_3 = |\langle 1,1|out\rangle|^2 = 9/16$, and the total detection probability is $P_{tot} = 1/3 \times P_3 = 3/16$.
Note, that this answer is different from just calculating the total probability for particle having having either $S_y = +\hbar$ or $S_y = 0$: $P_{tot} = 1/3 \times |\langle y_{+1}|1,1\rangle|^2 \times |\langle 1,1|y_{+1}\rangle|^2 + 1/3 \times |\langle y_0|1,1\rangle|^2 \times |\langle 1,1|y_0\rangle|^2 = 1/48 + 1/12 = 5/48$, since in this case we assume we have which-way information, so the wavefunctions of two channels do not interfere.