Two-Slit Interference, One Photon at a Time
Operating Manual (expanded)

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I. Introduction

I.A. Wave-particle duality

Pick up any book about quantum mechanics and you're sure to read about 'wave-particle duality'. What is this mysterious ‘duality’, and why should we believe that it's a feature of the real world? This manual describes the TeachSpin apparatus, which makes the concept of duality as concrete as possible, by letting you encounter it with photons, the quanta of light.

This apparatus makes it possible for you to perform the famous two-slit interference experiment with light, even in the limit of light intensities so low that you can record the arrival of *individual photons* at the detector. And that brings up the apparent paradox that has motivated the concept of duality -- in the very interference experiment that makes possible the measurement of the wavelength of light, you will be seeing the arrival of the energy of light in particle-like quanta, in individual photon events. How can light act like waves and yet arrive as particles? This paradox has been used, by no less an authority than Richard Feynman, as the introduction to the fundamental issue of quantum mechanics:

> “In this chapter we shall tackle immediately the basic element of the mysterious behavior in its most strange form. We choose to examine a phenomenon which is impossible, *absolutely* impossible, to explain in any classical way, and which has in it the heart of quantum mechanics. In reality, it contains the *only* mystery. We cannot make the mystery go away by explaining how it works. We will just *tell* you how it works. In telling you how it works we will have told you about the basic peculiarities of all quantum mechanics.” [R. P. Feynman, R. B. Leighton, and M. Sands, *The Feynman Lectures on Physics*, vol. I, ch. 37, or vol. III, ch. 1 (Addison-Wesley, 1965)]

You should find and read either of these famous chapters in the *Feynman Lectures* that introduce the central features of quantum mechanics using the two-slit experiment as an example. Feynman notes that he discusses the experiment as if it were being done with particles with rest mass, such as electrons; he wrote in an era in which he was discussing a ‘thought experiment’. But since that time, the two-slit experiment really has been done with neutrons [see *Reviews of Modern Physics* **60**, 1067-1073 (1988)], and in this experiment, you too will translate a thought experiment into a real one, in this case with photons.

There are several technical advantages to the use of photons. They are easily produced and detected, using the ordinary tools of optics. They propagate freely in air, and require no vacuum system. They are electrically neutral, and thus interact neither with each other nor with ambient electric and magnetic fields. They are (at high enough levels) directly visible to the eye. Finally, their wavelengths are (for typical visible-light photons) of a much more convenient size than the wavelength of electrons or neutrons. All these technical advantages make it possible to perform even the single-photon version of the two-slit experiment in a tabletop-sized and affordable instrument.

II.B. Historical context

There is a rich historical background behind the experiment you are about to perform. You may recall that Isaac Newton first separated white light into its colors, and in the 1680's hypothesized that light was composed of 'corpuscles', supposed to possess some properties of particles. This view reigned until the 1800's, when Thomas Young first performed the two-slit experiment now known by his name. In this experiment he discovered a property of destructive interference, which seemed impossible to explain in terms of corpuscles, but is very naturally explained in terms of waves. His experiment not only suggested that such 'light waves' existed; it also provided a result that could be used to determine the wavelength of light, measured in familiar units. Light waves became even more acceptable with dynamical theories of light, such as Fresnel's and Maxwell's, in the 19th century, until it seemed that the wave theory of light was incontrovertible.

And yet the discovery of the photoelectric effect, and its explanation in terms of light quanta by Einstein, threw the matter into dispute again. The explanations of blackbody radiation, of the photoelectric effect, and of the Compton effect seemed to point to the existence of 'photons', quanta of light that possessed definite and indivisible amounts of energy and momentum. These are very satisfactory explanations so far as they go, but they throw into question the destructive-interference explanation of Young's experiment. Does light have a dual nature, of waves and of particles? And if experiments force us to suppose that it does, how does the light *know* when to behave according to each of its natures? These are the sorts of questions that lend a somewhat mystical air to the concept of duality.

Of course the deeper worry is that the properties of light might be not merely mysterious, but in some sense self-contradictory. You will be confronted with just the sort of evidence, which has led some persons to worry that either light, or our theories for light, are not only surprising but also inconsistent or incoherent. As you explore the phenomena, keep telling yourself that the light is doing what light does naturally, and keep asking yourself if the difficulties lie with light, or with our theories of light, or with our verbal, pictorial, or even mechanical interpretations of these theories.

I.C. Goals for this apparatus

It is the purpose of this experimental apparatus to make the phenomenon of light interference as concrete as possible, and to give you the hands-on familiarity which will allow you to confront duality in a precise and definite way. When you have finished, you might not fully understand the *mechanism* of duality -- Feynman asserts that nobody really does -- but you will certainly have direct experience of the actual phenomena that motivate all this discussion.

Here, then, are the goals of the experiments that this apparatus makes possible:

1. You will be seeing two-slit interference *visually*, by opening up an apparatus and seeing the exact arrangements of light sources and apertures which operate to produce an 'interference pattern'. You'll be able to examine every part of the apparatus, and make all the measurements you'll need for theoretical modeling.
2. You will be able to perform the two-slit experiment *quantitatively*, recreating not only Young's measurement of the wavelength of light, but also getting detailed information about intensities in a two-slit interference pattern which can be compared to predictions of wave theories of light.
3. You will be able to perform the two-slit experiment *one photon at a time*, continuing the same kind of experiments, but now at a light level so low that you can assure yourself that there is at most one relevant photon in the apparatus at any time. Not only will this familiarize you with single-photon detection technology, it will also show you that however two-slit interference is to be explained, it must be explained in terms that can apply to single photons. [And how can a single photon involve itself with two slits?]
4. You will be exploring a pair of theoretical models, which attempt, at differing levels of sophistication, to describe your experimental findings. You will thus encounter the distinctions between Fraunhofer and Fresnel diffraction theories in a concrete case, and you will also learn the difference between a mathematical, and a physical, description of what is going on.

II. Introduction to the Apparatus

II.A. Familiarization with the Apparatus

This version of the manual assumes that the apparatus is already unpacked and assembled, but that it has not been brought into operation, or that it needs to have its optical alignment checked.

The apparatus consists of a long rectangular metal assembly, with a single-photon detection box attached to one end. Orient the long assembly on its wooden feet so that the box is at the right hand end of the assembly, and you'll be properly oriented to match the parts of the apparatus with the descriptions below.

First (before plugging anything in, or turning anything on) you'll need to confirm that the shutter, which protects the amazingly sensitive single-photon detector, is *closed*. Locate the detector box at the right end of the apparatus, and find the rod which projects out of the top of its interface with the long assembly. Be sure that this rod is pushed all the way *down*; take this opportunity to try pulling it vertically upward by about 2 cm, but then ensure that it's returned to its fully down position. Also take this occasion to confirm, on the detector box, that the toggle switch in the HIGH-VOLTAGE section is turned *off*, and that the 10-turn dial near it is set to 0.00, fully counter-clockwise.

Now it's safe for you to open the cover of the long two-slit assembly. Before you can do this, you'll need to open the four latches that hold it closed; execute a slight lift and a quarter-turn of each of these latches until they are visibly disengaged. The cover is still light tight and rather snugly closed, so you may want to lift it off by screwing *in* the brass thumbscrew at the far-left end of the cover. Then lift (by a cm or more) the far-left end of the cover of the apparatus. Now slide the whole cover sideways and leftwards by a cm or more; this will disengage the *right* end of the cover from its light-tight slot, so you can lift the whole cover off. [Take this opportunity to learn how to re-install the cover, making sure that you can engage its right end, lower its left end, back off the thumbscrew, and re-engage the four latches which hold it in place.]

With the cover open, you are ready to look over all the parts of the apparatus:

- At the left end are two distinct light sources, one a red laser and the other a green-filtered light bulb; also at the left end are the controls for the light sources.
- Found along the length of the long box are the various slits and apertures that form a Young's two-slit experiment. On the front of the box are two 'micrometer drives', which allow you to make mechanical adjustments to the two-slit apparatus.
- Finally, there are two distinct light detectors in the box at the right-hand end of the apparatus:
  * one is called the 'photodiode', which is used with the much brighter laser light source; it's attached to the light shutter in such a way that it's *in position to use* when the shutter is in its *down* position.
  * the other is called the 'photomultiplier tube' or PMT for short, which is used with the much dimmer light-bulb source. **It is safe to use *only* when the cover of the apparatus is in place, and the light bulb is in use; it is exposed to light only when the shutter is in its *up* position.**

Finally, there are electrical connections. Electric power comes from a power module, plugged into your AC line and connected by a cable to the left end of the apparatus. Two more cables run from this end to the detector box, and supply the photodiode and PMT detectors with the power they need for operation.

II.B. Alignment of the Apparatus

Before you take on the task of aligning the apparatus, it is worth examining all the slits. Each slit is stamped in a metal foil, whose edge is attached to a magnetic fixture that allows it to be placed conveniently in a slit-holder in the apparatus. Practice taking the source slit right off of its holder, and look it over with a magnifying glass. You’re looking at a single slit, and you’ll want to know its width; this can be measured by single-slit diffraction using a separate tabletop set-up and a HeNe laser, or much more directly by viewing the slit in a low-power microscope whose sample holder is equipped with a traveling stage.

By either technique, you can also characterize the detector slit, and the wide slit whose two edges are used to execute the slit-blocking functions. Finally, examine one or more of the double-slit assemblies that are available; for these you will want not only to measure the width of the two individual slits, but also to characterize their separation. However you measure this separation, it is conventional to characterize a double slit by the center-to-center separation of the two slits.

As you replace the slits, you'll note that they can be translated both vertically and horizontally to a limited extent; it is also possible to rotate them to some extent in their own plane. For best results, you want all the slits to have their long edges set to be accurately vertical; for the moment, use an 'eyeball accuracy' criterion for this.

Now it's time to align the whole apparatus; the goal will be an to get the apparatus aligned well enough so that you can switch from laser to light-bulb optical source without needing to disturb anything at all in the whole arrangement of slits. Here's the procedure:

1) Ensure that the PMT shutter is closed, and the PMT bias is turned off (by toggle switch) and down (to 0.00 on the 10-turn dial). Now connect the line cord to AC power, and learn how to turn on the laser source, and the light-bulb source -- use a paper card to see each source's output beam.

2) Turn the laser beam on, and now learn how to move the laser module between two positions:

- pulled forward toward you, it lies centered on the central axis of the instrument.
- pushed against the back wall of the apparatus, it clears the path for the light-bulb source to come into action.

In moving the laser module, you're feeling it attracted magnetically to the block across which it's moving.

3) Now you want to align the laser beam in the apparatus, and here's the easiest way:

- slide the laser module into its central-axis position.
- remove all four slit assemblies, and follow the laser beam, with a card, from its source along the full length of the apparatus.
- the goal is to get the beam centered on the detector-slit aperture, so put a viewing card there.
- find the brass thumbscrew on the underside of the apparatus, below the laser, and loosen it.
- now you can rotate the block holding the laser module about a vertical axis; rotate it so the laser beam is aimed correctly, and then tighten the thumbscrew you loosened.
- confirm that the laser beam ended up where you want it -- centered on the aperture where the detector slit will be mounted.
- if the laser beam's position needs vertical adjustment (as well as the horizontal adjustment you just finished), this can be accomplished using a 5/64" ball-tipped Allen wrench, rotating the stainless-steel Allen screw found just behind the laser module, beneath the wiring bundle.

4) Now you can add the slits back into the apparatus:

- put the detector slit in first, and do your best to make the slit's edges vertical.
- put in the slit-blocker (the wide slit), and use a card just downstream of it to see the broad stripe of light that it permits to pass through.
- put in the double slit, and now find the two narrow vertical ribbons of light that come through it. (Adjust the slit-blocker, using its micrometer, to ensure that both ribbons will pass through to your card.)
- finally, put in the source slit. This will at first block the laser beam, so adjust the slit (not the laser) by lateral translation in its plane, until it is centered on the beam.

5) To do the final adjustment of the source slit's position, look at the upstream side of the double-slit fixture, and find the single-slit diffraction pattern projected there by light passing through the source slit. Do a by-hand fine-adjust of the source slit's location, until the central maximum of this single-slit diffraction pattern is centered on the double-slit structure.

You can confirm that you've done this optimally by placing a viewing card downstream of the double slits and slit-blocker, and checking that the source slit has been moved so as to maximize the brightness of the laser light emerging from the double slits in two bright vertical ribbons of light.

6) Now you have the laser beam properly aligned to the apparatus, and all the slits aligned to that beam. Next you want to align the light-bulb source:

- push the laser module out of its central-axis position. This opens the path for light from the bulb to reach the slits. [You may practice returning the laser to its central-axis position.]
- turn the laser off, dial the BULB POWER knob down to zero, turn the switch from OFF to BULB, and dial up the bulb power to about half scale.
- in a dimmed room, use a view-card at the light-bulb source's output to confirm that some weak (green) light is emerging.
- learn to remove the green filter from the bulb source, by pulling axially downstream on the cylindrical 'output snout' of the bulb source. The filter assembly will emerge to be removed.
- find the thumbscrew on the underside of the apparatus that's under the light-bulb source, and loosen it. Now temporarily turn the bulb-power knob to full scale, and in a darkened room, follow the white light downstream. You should be able to see a widening ribbon of light emerge from the source slit and head for the double-slit structure.
- rotate the bulb-source assembly, about a pivot point near its output end, by pushing laterally on the brass thumbscrew. Your goal is to position the light-bulb's filament laterally so as to send light through the entrance slit and reach the double slits. When you have gotten the bulb to where it needs to be, tighten the thumbscrew to fix the bulb's location.

- now you may turn the bulb power down to half scale, and re-install the green filter.

Notice that you have *not* changed the location of any of the slits during this process, so that you now have an optical line-up that can be used, without re-alignment of the slits, with either the laser or the bulb sources.

7) There is next an alignment task for the slit-blocker; your goal is to get its two long edges aligned to be vertical. You may start by removing the slit-blocker's (wide) slit from its magnetic mount; remember the slit-blocker is mounted on the micrometer-adjusted, flexure-supported moveable block just downstream of the double slits. The best diagnostic for the proper positioning of the slit-blocker's wide slit is to use the two ribbons of light emerging beyond the double slit when they are illuminated by the laser source. Find these ribbons on a paper card, and now re-install the wide slit on its mount. Use the micrometer to translate the wide slit laterally until both ribbons emerge, and now dial the micrometer until you see the knife-edge function of the slit-blocker in cutting off one of the ribbons of light. If the wide slit is in place with its edges not correctly vertical, this knife-edging of the ribbon will proceed not all at once, but bottom-to-top or top-to bottom. Hand-adjust the wide slit by small in-plane rotations on its magnetic holder until you achieve the desired all-at-once chopping-off of the ribbon of light.

8) The equivalent alignment task for the detector slit is to rotate it in its own plane, until its length is accurately parallel to the double-slit fringes that it is to scan. The best diagnostic for this is the contrast of the interference pattern that you will record in section III.B. below. So for now, just be sure that the detector slit is vertical to 'eyeball accuracy'.

II.C. Ancillary equipment needed for operation

There are a few more electrical functions of the apparatus that you should now encounter, and a few electrical tools that you will need to assemble for operation of the apparatus.

You have already operated the light-bulb source; it has an on-off switch and an intensity adjustment. You've also operated the laser source, thus far merely on or off. For some modes of operation, you might want to turn the laser on and off repeatedly, and not just by toggling its switch. For that purpose, there is a laser-modulation input connecter at the source end of the apparatus; it's used in the optional lock-in mode of detection described in Appendix B.

At the detector end of the apparatus, you'll need a digital multimeter for reading the output voltage that emerges from the photodiode-amplifier mode of detection. You can understand this signal by noting that the photodiode translates input optical power to output electrical current with a sensitivity of about 0.4 A/W [Exercise: show that the conversion of photons of red light to electron-hole pairs in a semiconductor with 100% efficiency would yield about 0.5 A of output current for an input of 1 Watt of optical power.], and then the photodiode amplifier translates photodiode current to output voltage with a conversion constant of 22×10^6 V/A. So if we see an output of 1.0 Volt, we can infer an input current of 4.5 x 10^{-8} A = 0.045 μA, and further infer an incident optical power of 11 x 10^{-8} W, or 0.11 μW, on the photodiode. The photodiode amplifier has a time constant of 0.4 ms, which sets the timescale for the response time of this detector system.

Also at the detector end of the system are the electronics for running the photomultiplier tube. A PMT requires a high-voltage power supply for its operation, and this is integrated into the socket of the PMT. There is a toggle switch to activate this supply, and a 10-turn dial to set the high voltage somewhere in the range 0 - 1000 V dc. There is also a pair of monitor outputs, at which a potential difference of 10^{-3} of the PMT 'bias' is available for inspection. You may use the approximate conversion constants of 1.00 turn of 10-turn dial = 0.10 V at monitor output = 100 V of PMT bias. The 'gain' of the phototube, or the number of multiplied electrons out per single photoelectron event, rises exponentially with this bias voltage, and reaches about 10^6 at full bias. You will not be seeing this charge pulse directly, since it is sent directly into a pulse amplifier and discriminator module. What you will need is a moderately fast digital 'scope to monitor the amplified pulses, and a TTL-compatible counter to count those charge pulses which activate the discriminator and lead to the generation of a single-shot, fixed-width output 'event' pulse.

If you wish to investigate the operation of the discriminator in detail, there is a provision to inject 'test pulses' into the pulse amplifier. To do this, follow the directions of Appendix C.

III. Operation of the Apparatus

This section of the manual will introduce you to the functions of the apparatus, leading you through three stages of understanding until you have seen two-slit interference, one photon at a time.

III.A. Visual mode of operation

For this mode of operation, you will be working with the cover of the apparatus open. You will need to use neither light detector, since you will be using your eyes as the only detector needed. You'll be using the laser as light source, and you'll find a supply of business cards or other small white paper screens to be convenient tools.

Use the controls at the left end of the apparatus to turn on the solid-state diode-laser light source -- it's contained within a black metal block. Use a paper screen to find its bright red output beam; the diode laser manufacturer asserts that its output wavelength is 670 ± 5 nm [or 0.670 ± 0.005 μm], and its output power is about 5 mW. [So long as you don't allow the full beam to fall directly into your eye, it presents no safety hazard.]

Now follow the laser beam until it reaches the entrance slit, or 'source slit', of the two-slit apparatus; this is a single slit, of height about 1 cm, but width of only 0.085 mm. If your apparatus is aligned, the slit will be neatly straddling the laser beam, so that a good fraction of the laser light will be passing through the slit -- use your paper card to see if this is so.

Light passing through this narrow source slit will undergo 'single-slit diffraction', and you can follow this process by moving your viewing card downstream. You will see the red beam spread out horizontally, reaching a width of about a cm by the time it reaches the middle of the apparatus.

In the middle of the apparatus is the holder for the double slit, which is a structure with two rectangular apertures, again each about a cm high, and each with width of only 0.085 mm, and with center-to-center separation of 0.356 mm. [All of the slit structures are mounted on magnetic holders so that they can be removed, examined, and re-positioned; if your apparatus is already aligned, it would be a pity to spoil this alignment by disturbing the slits now.] Instead, put your viewing card just downstream of the two-slit structure, and look (close up!) to see if you can observe the two ribbons of light, just a fraction of a millimeter apart, which emerge from the two slits.

[In fact, you have available to you three distinct two-slit assemblies; if you look at one of your spares, you'll see a number (14, 16, or 18) hand-written onto the metal film into which the double slits are stamped. This number is the nominal center-to-center separation of the two slits, given in units of 'mils' = 0.001" = 25.4 µm. So the slit separations available to you are nominally 0.356 mm, 0.406 mm, and 0.457 mm; which of these two-slit assemblies do you in fact have installed in the apparatus?]

This is your chance to understand the function of the slit-blocker, just downstream of the double slit system, which will allow you selectively to block the light coming from either of the two slits. A ‘micrometer screw’ on the front center of the apparatus controls this slit-blocker; rotating the micrometer's knob will move the slit-blocker laterally across the two ribbons of light emerging from the two-slit system. For the present, find a position for the micrometer adjustment that permits the two ribbons of light to emerge and continue rightwards in the apparatus.

Each of the narrow ribbons of light emerging from a slit will continue to diffract; follow these broadening ribbons downstream until they visibly overlap. By the time your viewing card reaches the right-hand end of the apparatus, the overlap will be nearly complete; and you'll see that the two overlapping ribbons of light combine to form a pattern of illumination displaying the celebrated ‘fringes’ named after Thomas Young. How would you characterize these ‘fringes’? Can you describe them qualitatively? Can you distinguish between your description of the phenomenon you see and your hypothesis for its cause?

Now position a viewing card at the downstream end so you can refer to it for a view of the fringes, and take another card back upstream to the vicinity of the slit-blocker. Learn to dial the slit-blocker's micrometer adjuster [the one at the *middle* of the apparatus] until you see how to use it to block the ribbon of light coming from either the farther, or the nearer, of the two slits. Start by rotating the multi-turn micrometer screw fully clockwise, and watch this adjustment push the slit-blocker away from you. Take this opportunity to learn how to read a micrometer dial -- see Appendix A if you haven't done this before -- and now, as you dial the micrometer counter-clockwise, find and record five settings for the slit-blocker's micrometer:

- one position for which both slits are blocked;
- another for which light emerges only from the farther of the two slits;
- a third (anywhere in a wide range) that allows both ribbons of light to emerge;
- a fourth for which light emerges only from the nearer of the two slits; and finally,
- a fifth setting (and highest reading) which again blocks the light from both slits.

It is essential that you are confident enough in your ability to read, and to set, these five positions that you'll be able to do so even when the box cover is closed (when you won't be able, as now, to confirm your results by checking with a white viewing card).

Once you've gotten these five settings, let the light reach your viewing card at the far-right end of the apparatus, and find out what happens there, qualitatively, for each of the five settings. In two of them, no light at all will arrive; in the third of them, light will arrive from both slits to form Young's two-slit interference fringes. What happens in the other two cases, when light from only one slit arrives?

In particular, observe what occurs at some *particular locations* on your screen:

- at a bright fringe, or 'interference maximum', what happens to the light intensity when you use the slit-blocker to cover one slit?
- at a dark fringe, or 'interference minimum', what happens to the light intensity *at that location* when you use the slit-blocker to cover one slit?

You should be able to use the language of interfering light waves to describe what you see, and to explain why it happens. You'll be investigating this behavior quantitatively in later parts of the experiment, but for now you need to be familiar with it qualitatively, and in terms of causes.

There's one last piece of the apparatus that you can now learn to use. At the far-right end of the apparatus is a final optical element; it's an exit slit, or 'detector slit', of the same size and character as the source slit, except that it’s mounted on a moveable structure like the slit-blocker, so it too can have its position adjusted by a micrometer screw drive. The purpose of this detector slit is to allow light from a narrow slice of the interference pattern to pass along to the end of the long apparatus and into the detector box. By translating the detector slit laterally along the interference pattern in space, you can select which part of the pattern will have its light sent on to the detector. Thus by scanning the micrometer screw of the detector slit, you can scan over the interference pattern, eventually mapping out its intensity distribution quantitatively. For now, ensure that the detector slit is located somewhere near the middle of the two-slit interference pattern.

You are now acquainted with every part of this version of Young's experiment, and with the two 'independent variables' you can control: one is the position of the slit-blocker, and you have recorded settings corresponding to each of five slit conditions; the other sets the location of the detector slit. Now you're ready to go on to quantitative measurements.

III.B. Quantitative mode of operation

This mode of operation of the apparatus continues to use the laser light source, but it begins to use the photodiode detector to survey quantitatively the intensity distribution of the interference pattern, by varying the position of the detector slit. You might conduct these measurements with the box cover open, but room light will contribute excessive and variable contributions to your signals, so now is the time either to dim the room lights or to close up the cover of the apparatus. For convenience, have the slit-blocker set to that previously determined setting which allows light from both slits to emerge and interfere.

The shutter of the detector box will still be in its closed, or down, position. This blocks any light from reaching the PMT, but the shutter in its down position correctly centers a 1-cm^2 solid-state 'photodiode', which acts just like a solar cell in actively generating electric current when it's illuminated. The device is equally sensitive everywhere over its area, so it would record *all* the light in the whole interference pattern if it were not for the detector slit. But with the detector slit at a fixed position, the only light reaching the detector is that from a selected part of the interference pattern; by this means, a single-element, spatially-fixed, large-area detector can serve to record (serially in time) the intensities at various places in the interference pattern. The method of course relies on the fact that the rest of the apparatus is stable in time; happily, the diode-laser source has an output power varying by <0.1% in time, and the mechanical stability of the rest of the apparatus is also adequate.

The electric current from the photodiode is conducted by a thin coaxial cable to the INPUT BNC connector of the photodiode-amplifier section of the detector box. At the OUTPUT BNC connector adjacent to it, there appears a voltage signal derived from that photocurrent. Connect to this output a digital multimeter set to 2 or 20-Volt sensitivity; you should see a stable positive reading.

To determine if this reading means anything, go back to the left end of the apparatus and use the 3-position toggle switch to turn the laser source off. This should reduce the voltage signal you've been seeing, but perhaps not to zero; record the value you see, and take it to be the 'zero offset' of the photodiode-detector system. You might turn off the room lights to confirm that the signal you see is actually an electronic offset, and not the leakage of light into your apparatus. The zero-offset reading will eventually need to be subtracted from all the other reading you make of this output voltage.

Turn your laser source back on, and now watch the photodiode's voltage-output signal as you vary the setting of the detector-slit micrometer. If all is well, you will see a systematic variation of the signal as you dial the micrometer; you are scanning over the interference pattern. You'll find a variety of maxima, and you should try to find the highest of the maxima -- this is the 'central fringe' or 'zeroth-order fringe' which theory predicts. Between the various maxima, you should see minima; and if your alignment is good, the signal at these minima should drop nearly to the zero-offset signal you previously recorded. These deep minima are of course the manifestation of destructive interference.

The size of the signals you observe will be somewhere in the vicinity of 5 Volts at the central maximum, but the number will depend a great deal on the details of your alignment procedure. If the (offset-corrected) signal is less than 1 Volt, you will probably want to improve the alignment. Here's a very approximate calculation that suggests why signals of order 1-5 Volts are to be expected. Start with 5 mW of laser power, and suppose that 2 mW of that makes it through the source slit. This power diffracts laterally to about 10 mm width, and thus one of the double slits, of width about 0.1 mm, can pass only about 1% of this, or 20 μW. This power in turn diffracts out to a stripe about 10 mm wide at the plane of the detector slit, which is turn can pass only about 1%, or 0.2 μW, to the photodiode detector. That yields about 0.1 μA of photodiode current, and hence about 2 Volts at the photodiode signal output. If we take into account not only one of the double slits, but also interference of light from both of them, we can understand why signals of order 1-5 Volts are to be expected. Nevertheless, these are very large signals in terms of rate of photon arrivals. [Exercise: compute the rate of arrival of red photons that will deliver energy at a rate of 0.2 μW.]

Before you go on to record data systematically, park the detector slit at the location of the central maximum, and then go over to the slit-blocker micrometer screw, and set it to a (previously determined) reading that you know will permit light to pass through only *one* of the slits. Check what photodiode signal you now are getting -- it should be less than before. Again, set the slit blocker to permit light only from the *other* of the two slits, and record another diminished signal. Can you explain why the signals have diminished? Corrected by subtraction of zero-offset, by what factor should they have diminished? [Answer: *Not* to 50%, but to a *smaller* fraction, of the original intensity -- why?]

To see another and even more dramatic manifestation of the wave nature of light, set the slit blocker again to permit light from both slits to pass along the apparatus, and now place the detector slit at either of the *minima* immediately adjacent to the central maximum; take some care to find the very bottom of this minimum. Now you're seeing the effect of destructive interference -- what will happen when you use the slit-blocker to block the light from one, or the other, of the two slits? [Answer: Blocking fully half the light coming through the two-slit assembly is going to *raise* the signal you're looking at -- to what level? and why?]

Once you have performed these spot-checks, and have understood the motivation for them, and the explanation of their results, you are ready to conduct systematic measurements of one dependent variable (the photodiode voltage-output signal) as a function of two independent variables. What you want are three graphs, each giving the voltage-output signal as a function of the detector-slit position; the graphs will be for one slit open, the other slit open, and both slits open. You will want occasionally to block both slits to get a measure of the zero-offset signal that needs to be subtracted from all the readings you take. You will learn, by trying, what spacing of detector-slit positions to use. You will learn this fastest if you, or your partner, plot the data as you take it -- nothing beats an emerging graph for teaching you what is going on.

The data you have obtained are along a calibrated horizontal scale -- if you read Appendix A, you will know what the readings of the detector-slit micrometer imply, quantitatively, for position along the interference pattern. The data are also obtained on a quantitative vertical scale, though this one is in 'arbitrary units' -- we have not translated voltage-output units into light-intensity units, but you may be assured that the (offset-corrected) signal you obtain is linear in light intensity. Thus your data can be directly compared with theoretical models of single-slit diffraction and two-slit interference; section IV of this manual discusses two models that can be used to explain your data. In particular, the spacing of your interference minima and maxima gives a quite direct measure of the wavelength of the red laser light you are using.

The data you obtain also serve as another diagnostic of alignment procedures. You have data plotted for both cases of one-slit-blocked, and these should display characteristic single-slit diffraction features. In particular, the heights of these two graphs should be equal; if they are not, it could be that the source slit is unequally illuminating the two slits of the double-slit assembly. You also have data for the Young's-experiment two-slit-interference case, and the depth of the minima of this curve is another searching test of alignment. With moderate care, you can achieve a contrast ratio,

(offset-corrected central-maximum signal)/
(offset-corrected first-minimum signal),

of over 30-to-1. If your contrast ratio is markedly worse, you might wonder if the double slits are being unequally illuminated, if stray light is reaching your detector (which would wash out the contrast), or if the scanning detector slit might fail to have its long dimension carefully parallel to the long straight interference fringes. You want the detector slit to be oriented to permit an alignment error of only a fraction of its width, say 0.05 mm, over the 10 mm of fringe height that can illuminate the detector. This means the fringe contrast is sensitive to a rotation of the detector slit in its own plane by less than 1°.

Since the slit-blocker provides a way to give the signal when light from neither slit is reaching the detector directly, you have a fine operational method for finding the background level that should be subtracted from the data. Then you may be guided by theory to expect, with the detector slit parked at the central maximum, background-corrected signals in the proportions 1:4:1 for the use of one:both:other of the two slits. You may also be guided by theory to expect, with the detector slit parked at the locations of the innermost minima, background-corrected signals in the proportions 1:ε:1 for the use of one:both:other of the two slits; here ε is related to the inverse of the contrast ratio previously discussed. There's no need to obsess yet about departures from these theoretical ideals, since they apply only in certain limiting cases; rather, you should compare your results to the expectations from a corpuscular view of light, in which the addition of 1 unit of light from one slit, and 1 unit of light from the other slit, ought always to yield 2 units of light. If instead you have displayed a result much nearer to 4 than to 2, you have falsified some corpuscular views. More dramatically still, you have shown that the addition of 1 unit to 1 unit not only can yield a result smaller than 2, it can yield a result markedly smaller than 1! This is naturally explained by a wave theory, but you should put into words in just what sense it is evidence against a corpuscular view of light.

III.C. Single-photon mode of operation

If you have gotten the visual and quantitative modes of operation of this apparatus to work, you are ready for this section; but if you have not found interference fringes, trying single-photon detection is not going to cure your problems. So this section assumes

- that you know how to use the slit-blocker, and have left it in position to pass light from both slits; and
- that you know how to find the central maximum of the interference pattern, and have left the detector slit parked to pass light from that central fringe to the detector box.

Now you're ready to go on to the use of the photomultiplier tube, or PMT, which makes available to you electrical pulses that correspond to the detection of light, one photon at a time.

But first a **WARNING**: a photomultiplier tube is so sensitive a device that it should not be exposed even to moderate levels of light when turned off, and must not be exposed to anything but the dimmest of lights when turned on. In this context, ordinary room light is intolerably bright even to a PMT turned off, and light as dim as moonlight is much too bright for a PMT turned on. That is why the PMT box is equipped with a shutter, and the apparatus as a whole is equipped with a buzzer alarm, to help protect the PMT from misadventure. The goal of these safety measures is to assure that the PMT is used

- *only* when its box is coupled to the two-slit apparatus, and
- *only* when the cover of the apparatus is closed, and
- *only* when the light-bulb (and ***not*** the laser) source is being used inside the box.

[Exercise: To understand the implications of light leaks, and to use some numbers typical of the PMT in this apparatus, suppose that ‘full sunlight’ delivers 10^3 W/m^2 to the earth, and that ‘full moonlight’ is a million times dimmer. Suppose that this moonlight is delivered by photons of yellow light, that they fall on a photocathode of area 8 x 25 mm^2, and that they are converted to photoelectrons with efficiency 4%. Suppose finally that each photoelectron emitted at the photocathode is amplified to a pulse of 10^6 electrons arriving at the anode. Calculate: a) the arrival rate of photons at the photocathode, b) the emission rate of photoelectrons, c) the arrival rate of electrons at the anode, and d) the average anode current this represents; finally, compare that current with the manufacturer’s suggested limit of 1 μA for average anode current.]

Before you do anything with the PMT, locate on the detector box the HIGH-VOLTAGE toggle switch which activates its power supply, and ensure that it's turned off; also turn down to 0.00 the 10-turn dial which will later set the voltage that enables the process of electron-multiplication inside it. Also, ensure that the shutter of the box is still in its *down* position.

Now it's both safe and necessary to open the cover of the apparatus, because you're about to change from laser to light-bulb illumination of the two-slit apparatus. When you open the cover, you might find the laser still running; use the 3-position toggle switch to turn it off. You'll need to slide the laser source (out of the path of light from the bulb source to the source slit) by pushing it away from you, against the back wall of the apparatus. Now turn the bulb power adjustment dial down to 0 on its scale, and set the 3-position toggle switch to the BULB position; dial the bulb adjustment up from 0 until you see the bulb light up.

The humble #387 flashlight bulb you're using will live longest if you minimize the time you spend with it dialed above 6 on its scale, and if you toggle its power switch only when the dial is set to low values.

If the apparatus has been aligned, the bulb should now be in position to send light through the apparatus. To confirm that the bulb is in the correct position, you will need to be able to darken the room completely, and you'll need a dim flashlight and a white paper viewing card. To make the bulb's output more visible, temporarily remove the green filter from its output end. Set the bulb's brightness to about half scale, and follow its splatter of white-light emission to the source slit. Now find the narrow ribbon of light which emerges downstream of the source slit, and trace it along the apparatus in the direction of the double-slit structure. As you travel downstream, the white band will widen horizontally, not only because of diffraction, but also because the filament of the light bulb is extended a few mm horizontally, and the source slit is acting like a one-dimensional pinhole camera. So the ribbon of light will be about a cm wide when it reaches the double-slit holder; what you need to ensure is that this cm-wide stripe of light is on-center in the apparatus and is thus falling on the double-slit structure. If the cm-wide ribbon is off-center and entirely missing the slits, you will need to go through the alignment procedure discussed in section II.B. of this manual. But if the ribbon illuminates both slits, you're all set; it is not necessary that it be perfectly centered in the apparatus.

Now put the green filter-holding structure back into the right-hand, downstream end of the light-bulb source. The green filter blocks nearly all the light emerging from the bulb, allowing passing only wavelengths in the range 541 to 551 nm. This is a small fraction of the total light, and while you might be able to see a ribbon of green light emerging from the source slit, you will probably not be able to follow it very far downstream. No matter; plenty of green-light photons will still be reaching the double-slit structure -- in fact, you should now dim the bulb even *more*, by setting its intensity control down to about 3 on its dial.

[Exercise: To see why you need to turn down the light bulb, and to gain familiarity with intensities expressed in units of photons/second, try to estimate -- to the nearest order of magnitude -- some photon transport rates:

First, assume a #378 bulb runs at 6.0 V, 0.2 A, and converts electrical energy into light with 5% efficiency. Further suppose that it puts out (on average) photons of red light. What is the total production rate of photons?

Second, assume that the output spectrum is in fact spread over the range 500 to 1500 nm, and that the green filter passes only a 10-nm bandwidth; what is the production rate of photons that would pass through the green filter?

Third, suppose these photons are emitted in all directions, but that all are absorbed except for those which reach a slit of size 0.1 x 10 mm^2, located about 100 mm from the bulb; what is the rate at which green photons will pass through this entrance slit?

Fourth, suppose that photons passing through the entrance slit spread to cover a 1 cm^2 area by the time they reach the double slits; what is the rate at which green photons pass through the double slits?

Fifth, suppose that photons passing through the double slits diffract to cover a 1 cm^2 area by the time they reach the detector slit; what is the rate at which green photons pass through the detector slit?

Finally, given the PMT efficiency of about 4%, what rate of photon events would result from all these assumptions?

You should find that further dimming of the bulb is necessary; that's why the apparatus makes it possible to adjust the bulb's intensity. If we turn it down to half voltage, it gets about half the current, hence 1/4 the power; more importantly, the bulb cools off and the peak of its blackbody spectrum moves toward the infrared. The fraction which passes the green filter, on the short-wavelength side of the blackbody peak, plummets exponentially for lower bulb temperatures, so it's very easy to reduce the green emission not just by 10-fold but by a factor of 10^3 or more. The result is that it's feasible to achieve the enormous dilution of photon count rate down to the level desired.]

You may now return to ordinary room illumination, and you **must** close up the cover of the box. Be sure that first the right end, and then the left end, of the cover are fully engaged, and then that all four latches are in place. Now, and *only* now, are you ready to start activating the photomultiplier tube (PMT) to detect the photons that are flying through the box.

You'll need a reasonably fast (>20 MHz bandwidth), preferably digital, oscilloscope for first examination, and a digital counter sensitive to TTL-level (+4 V positive-going) pulses for eventual counting, of photon events. Set the 'scope to about 50 mV/division vertically, and 200 ns/division horizontally, and set it to trigger on positive-going pulses of perhaps >20 mV height. Now find the PHOTOMULTIPLIER OUTPUT of the detector box, and connect it via a BNC cable to the vertical input of the 'scope; use a 50-**Ω** termination at the 'scope. You are about to look at pulses, each of which starts with the light-induced ejection of a single electron from the light sensitive photocathode of the PMT. Those photoelectrons will be amplified by about 10^5 inside the PMT, and arrive as a pulse of about 10^5 electrons, or about -2×10^{-14} Coulombs, at the input of a charge-sensitive amplifier. That device will convert that pulse of negative charge to a positive-going voltage pulse, which you will see on the 'scope.

When you are ready with these electronics, keep the shutter closed, set the HIGH-VOLTAGE 10-turn dial to 0.00, and turn on the HIGH-VOLTAGE toggle switch. You are now ready to start dialing up the high-voltage supply which provides the potentials inside the PMT needed for the electron-multiplication process; watch the 'scope display as you start to dial up the voltage, about a turn at a time. [Each full turn raises the PMT 'bias' by about 100 Volts, and the 'gain' of the PMT grows exponentially with this voltage setting. A pair of terminals on the panel allows you to monitor the potential difference (PMT bias/1000) using an ordinary digital multimeter.] As you raise the bias voltage, you will see occasional transient electronic noise, but the 'scope display should be quiet at each steady voltage. Somewhere around a setting of 4 or 5 turns of the dial, you should get occasional positive-going pulses on the 'scope, occurring at a modest rate of 1-10 per second.

- If you see some sinusoidal modulation of a few mV amplitude, and of about 200 kHz frequency, in the baseline of the PMT signal, this is normal.
- If you see a continuing high rate (>10 kHz) of pulses from the PMT, this is *not* normal, and you should turn down, or off, the bias level and start fresh -- you may have a malfunction, or a light leak.)

If you see this low rate of pulses, you have discovered the 'dark rate' of the PMT, its output pulse rate even in the total absence of light. You also now have the PMT ready to look at photons from your two-slit apparatus, so finally you may open the shutter (by grasping that vertically-emerging rod and lifting it a few cm). The 'scope should now show a much greater rate of pulses, perhaps of order 10^3 per second, and that rate should vary systematically with the setting of the bulb intensity.

If you can see those analog pulses, you are ready to look at the digital pulses they trigger. Inside the PMT pulse amplifier is a pulse generator, which emits, at the OUTPUT TTL connector, a single pulse, of fixed height and duration, each time the analog pulse you're viewing exceeds an adjustable threshold. Use another BNC cable to send such pulses to a second vertical channel of your 'scope, this one set for 2 V/div vertically. Now start with the discriminator control on your PMT box set to 0, and watch the 'scope display the analog, and TTL-level, pulse outputs in dual-trace operation. You should be able to find a discriminator setting, low on the dial, for which the 'scope shows one TTL pulse for each of, and for only, those analog pulses which reach (say) a +50-mV level.

- If your analog pulses are mostly not this high, you can raise the PMT bias by half a turn (50 Volts) to gain more electron multiplication.
- If your TTL pulses come much more frequently than the analog pulses, set the discriminator dial lower on its scale.

Now send the TTL pulses to a counter, arranged to display successive readings of the number of pulses that occur in successive 1-second time intervals. These represent photon-induced events (which are converted in the PMT to photoelectrons, then multiplied to electron pulses, then amplified to voltage pulses meeting a threshold criterion). *Because of the <100% efficiency of the PMT, not every photon is detected; but you are seeing a representative subset of events due to the arrivals of individual photons.*

To confirm that this is true, record a series of 'dark counts' obtained with the light bulb dialed all the way down to 0 on its scale. Now choose a setting that gives an adequate photon count rate (about 10^3/second) and use the slit-blocker, according to your previously obtained settings, to block the light from both slits. This should reduce the count rate to a background rate, probably somewhat higher than the dark rate. Next, open up both slits, and try moving the detector slit to see if you can see interference fringes in the photon count rate. You will need to pick an detector-slit location, then wait for a second or more, then read the photon count in one or more 1-second intervals, before trying a new detector-slit location. If you can see maxima and minima, you are ready to take data.

A first sort of data will enable you to be assured that you have set the PMT bias voltage to a suitable range. For this data, the independent variable is the PMT voltage setting, or one of its surrogates, the 10-turn dial setting (1.00 turn = 100 V of bias) or the monitor voltage (0.10 V = 100 V of bias). The dependent variables are both count rates as read by the TTL counter: one is the count rate at the central maximum of the interference pattern, with both slits open; the other is the dark rate, the count rate obtained under identical conditions but with the PMT shutter closed. Try recording both rates over the range 300 - 650 V of PMT bias, and plot the two count rates on a semi-logarithmic graph. You should see the 'light rate' reach a plateau, with the interpretation that you have reached a PMT bias which suffices to allow each genuine photoelectron to trigger the whole chain of electronics all the way to the TTL counter; you should also see the (much lower) 'dark rate' also rising with PMT bias. See if your graph motivates a setting at which you are counting substantially all of the honest-to-goodness photon events, but minimizing the number of 'dark events'.

Hereafter you may leave the PMT bias fixed, and you may set the bulb intensity to yield some convenient count rate (10^3 - 10^4 events/second) at the central maximum. You will now note that you seem to be getting an 'unstable' count rate, with apparent fluctuations -- this is in contrast to the rock-steady signal that you previously got in the quantitative mode of operation in III.B. In fact, the fluctuations you are seeing are a necessary consequence of the independent arrival of photons, and the fluctuations are expected to obey a statistical law. So with everything else stable, take a dozen or more successive readings of the number of counts in 1-second interval; call these samples N_i. The list $\{N_i\}$ has a mean value, N_{avg}, calculated in the ordinary way; the list also has a standard deviation, σ_{n-1}, which measures the size of the typical deviation from the mean. Compute these numbers for your list, and now compare with the prediction of 'photon statistics' which claims $\sigma_{n-1} = (N_{avg})^{1/2}$.

By changing the light bulb's intensity, you can easily arrange to make N_{avg} vary from order 10^2 to 10^4 for 1-second count intervals; if you take a list of data $\{N_i\}$ for each of several bulb settings, you'll be able to test whether σ_{n-1} of each list really does behave as $(N_{avg})^{1/2}$. You might try a log-log plot of σ_{n-1} as a function of N_{avg}, and look for a power-law fit of the form y = A x^B = (1.0) x^(0.5); note there are *no* free parameters in this prediction from photon statistics.

With this sort of evidence that you are detecting the independent arrivals of individual photons at your detector, you are now ready to take just the same sort of data as in part III.B. of this manual, except that now the dependent variable is photon count rate. You'll record this as a function of detector-slit position, making records for three cases: one slit, the other slit, and both slits open. You'll also need to make occasional readings with both slits blocked, to establish a 'background rate' of photons reaching the detector but not passing through either of the two slits.

Your graphs will again have a calibrated horizontal axis of known scale; this time their vertical axis will be in absolute units, of photon events detected per unit time. It is perfectly true that the PMT is not 100% efficient; in fact, for green light, it produces countable electronic events for only about 4% of the incident photons. Now take your central-maximum reading for the event rate, and use this efficiency estimate to infer the rate of arrival of all photons at the photocathode of the PMT. From this arrival rate, compute the average time interval between the arrivals of successive photons. Next, compute the ‘time of flight’ of a photon in the apparatus, the time it takes to traverse the distance from the bulb source to the detector. Comparing the time of flight of a given photon to the typical time interval between the arrivals of successive photons, you can compute the fraction of the time there is even one photon in flight through the apparatus. What is this fraction for your data? Since you've shown the photon arrivals follow the statistics of independent events, the probability that there are ever *two* relevant photons in the box is truly tiny, of the order of the *square* of this probability. This is the sense in which this apparatus allows you to work 'one photon at a time'; and yet even at this low rate of photons, you can see phenomena that you attribute to constructive and destructive interference. The agonizing question is -- interference of what, with what?

When you have graphed your data, you'll be able to see a number of new phenomena. First of all, from the spacing of the interference maxima, you'll be able to see immediately that the light in question has a different wavelength than the red laser light you used previously. Second, you'll be able to find detector locations at which you can *raise* the photon arrival rate by *closing* one of the slits. Here you'll be in the closest possible contact with the central question of quantum mechanics: how can light, which so clearly propagates as a wave that we can (with this very apparatus) measure its wavelength, also be detected as individual photon events? Or alternatively, how can individual photons in flight through this apparatus nevertheless 'know' whether one, or both, slits are open, in the sense of giving photon arrival rates which *decrease* when a second slit is opened? A frank oral dialogue between advocates of 'wave-nature' and of 'particle-nature' for light, each using experimental evidence to poke holes in the other's assertions, will do a great deal to teach participants why concepts as slippery as duality have had to be invented.

IV. Theoretical Modeling for the Experiments

IV.A. Fraunhofer Models for Interference and Diffraction

The simplest models for the data you have now taken are based on the assumptions of Fraunhofer diffraction, and are described in generic textbooks on electromagnetism. Find a reference you like, and now compare the assumptions of the theory with the facts of your experiment:

Theory assumes light reaches the double-slit assembly as a plane wave, ie. from a source infinitely far away; your light reaches the double slits from the source slit, about 38 cm away.

Theory assumes the double slits are the same width (call it *a*) and that they have a center-to center separation (call it *d*); theory also assumes a 2-dimensional approximation, with the slit lengths indefinite. Your apparatus has slits with approximate values $a = 0.085$ mm, $d = 0.353$ mm, and slit lengths of about 10 mm.

Theory assumes the light is of a definite wavelength λ; in your experiment, the laser light is monochromatic, with λ somewhere in the range 0.670 ± 0.005 µm, while the filtered bulb light is not monochromatic, but is distributed over a range from about 0.541 to 0.551 µm.

Theory assumes the light spreads from the double-slit assembly toward a point detector also infinitely far away, so the radiation pattern can be expressed in terms of an angular variable θ, measured in radians away from the central axis of the apparatus. Your detector is not point-like, but is effectively 0.085 mm wide; nor is it infinitely far away, but a distance of about 50 cm downstream. Nevertheless, it is approximately measuring light intensity as a function of an angular variable; make a model that relates the theoretical parameter θ to the detector-slit position you can set with its micrometer.

Now find a reference that gives a predicted intensity distribution analogous to

$$I(\theta) = I_0 (\cos\beta)^2 \left(\frac{\sin\alpha}{\alpha}\right)^2 \text{; where } \alpha = \frac{\pi a}{\lambda}\sin\theta \text{ and } \beta = \frac{\pi d}{\lambda}\sin\theta .$$

Get some practice graphing this expression, until you understand the role of the two factors in the intensity expression; also, get familiar enough with its derivation that you can understand how the theoretical prediction changes when one, or the other, of the two slits is blocked.

The theory doesn't pretend to give the intensity I_0 at central maximum, whether that's in units of photodiode voltage or photon count rate; so you can supply this value to the theory 'by hand'. The theory also can't know what setting of your detector-slit micrometer corresponds to the location of the 2-slit pattern's central maximum, so you can provide this value 'by hand' as well. But with these vertical-scale and horizontal-zero adjustments, you should be able to inter-compare theoretical predictions and experimental data for signals as a function of detector slit position.

You could even indulge in a bit of optimization:

- Does your model predict the fringes of the right height? Change the central-intensity parameter.

- Does your model predict fringes centered about the same center of symmetry as your data shows? Change your central-location parameter.
- Does your model display fringes of the right spacing? Change your slit-spacing parameter, or (if you're sure of the *d*-value), your wavelength parameter.
- Does your model display a fringe envelope of the right character? (Look at the intensities of the maxima other than the central maximum.) Try changing your slit-width parameter.
- Does your model include the effects of a distribution of wavelength values? Test the effect of including a range of wavelengths appropriate to your experiment.
- Does you model include the effect of the widths of the source and detector slits? Try to model at least the nonzero width of the detector slit.

By these tests, you will not only learn how the model's predictions depend on its input parameters, you will also learn how well your data constrain these parameters' values.

You also have taken data for signals obtained with one, or the other, of the two slits closed. What does your Fraunhofer theory predict for these signals? Note that there are now no 'free parameters' at all, since your two-slit data have determined all of them; so overlay the predictions on your data, and have a look at one of the deficiencies of a Fraunhofer model.

IV.B. Fresnel and Other Models for Interference and Diffraction

There are various deficiencies of the simple Fraunhofer models discussed above, some of which show up directly in comparisons with experimental data. This section suggests a more general method for modeling interference and diffraction phenomena, one which is free of some of these limitations; in particular, in does not require the assumption that source and detector are located ‘at infinity’. Even so, it is still not a fundamental electromagnetic calculation, since it fails to consider the polarization of actual electromagnetic fields.

Advanced electromagnetism textbooks suggest a variety of methods for treating diffraction, some based on Huyghen’s principle for wave propagation, and some based on Kirchhoff’s integral for scalar wave fields. Both of these theoretical approaches reveal the crucial role of phase, and particularly of the phase variation of a wave field with respect to position. So crucial is this role of phase that nearly all the features of light’s behavior can be captured in a truly remarkable model described in Feynman’s book *QED: the strange theory of light and matter*. Is this book, Feynman models the behavior of light by considering *only* its phase variation, taken to depend on positional displacement Δs according to the complex factor

$$\exp[2 \pi i \Delta s / \lambda]$$

for a monochromatic wave field whose free-space wavelength is λ. More surprisingly, Feynman uses the ‘sum over paths’ approach, and given light source at P, assigns to another location Q a wave disturbance that is just the (complex) sum of the results computed along *any and all imaginable* paths that lead from P to Q. Along each path, the overall phase is taken to be the product of all the phase factors for each infinitesimal part of the path. Finally, he supposes that a physical observable, such as the probability of detecting a photon at location Q, is given by the absolute square of the complex amplitude computed by the sum-over-paths approach.

Independent of the justification of this model (which emerges naturally from the Feynman picture of quantum field theory), it provides a readily calculated model for interference phenomena, particularly if we make the daring assumptions that we may use an essentially two-dimensional treatment of the apparatus, and that we may consider only those paths which proceed in straight lines from one slit to another. With these approximations, we can locate slits along a longitudinal central axis, with separation D_1 from source-slit plane to double-slit plane, and further separation D_2 from double-slit plane to detector-slit plane. We can also introduce locations P, Q, and R in the apertures of the source, double, and detector slits, respectively, located off the central axis by horizontal displacements x, y, and z. (In the spirit of the 2-d approximation, we ignore altogether the vertical coordinates of P, Q, and R.) Then we compute the phase factors for the (assumed straight-line) paths from P to Q, and for Q to R, according to displacements

$$s_1 = [D_1{}^2 + (x - y)^2]^{1/2} \quad \text{and} \quad s_2 = [D_2{}^2 + (y - z)^2]^{1/2} \quad .$$

For a path from source-point P to detector-point R, via intermediate point Q in the double-slit plane, we get an overall phase factor

$$\exp[2 \pi i s_1(x, y) / \lambda] \cdot \exp[2 \pi i s_2(y, z) / \lambda]$$

and in the spirit of the sum-over-paths approach, we take the overall field disturbance at R to be the sum (here, the integral) over all possible locations for Q. This yields the integral

$$I(x, z) = \int_{y \in S} dy \exp[2 \pi i s_1(x, y) / \lambda] \cdot \exp[2 \pi i s_2(y, z) / \lambda]$$

where the domain of integration S is the combination of the two slits: S_1 ranging from $(-d/2 - a/2)$ to $(-d/2 + a/2)$, and S_2 ranging from $(d/2 - a/2)$ to $(d/2 + a/2)$. Finally, the absolute square of this integral is taken to be an observable signal, corresponding to the photon detection rate for point detector at z, given point source at x.

The experimental apparatus in fact has source locations spread from $x = -a/2$ to $x = +a/2$, and detector locations spread from $z = t - a/2$ to $z = t + a/2$ (where t gives the location, relative to the central axis, of the center of the detector slit), so final integrations over x and z will account for the nonzero widths of the source and detector slits.

The computational burden of this approach can be reduced by series expansions of the path lengths s_1 and s_2, in powers of the (rather small) off-axis distances x, y, and z; for example,

$$s_1(x, y) = D_1 \left[1 + \left(\frac{x - y}{D_1}\right)^2\right]^{1/2} = D_1 \left[1 + \frac{1}{2} \left(\frac{x - y}{D_1}\right)^2 + \ldots\right]$$

Taking only the lowest-order (quadratic) terms in the series reproduces the Fresnel approximation in optics, and can be justified for the parameters appropriate to this experiment. [Exercise: compute a typical next-order term, and show that even when exponentiated, it yields only a negligible correction.] This approximation also allows two factors to be removed from the integral above, giving

$$I(x, z) = \exp[2\pi i\, D_1/\lambda] \cdot \exp[2\pi i\, D_2/\lambda] \int_{y \in S} dy\, \exp[2\pi i \frac{(x - y)^2}{2\, D_1\, \lambda}] \cdot \exp[2\pi i \frac{(y - z)^2}{2\, D_2\, \lambda}]$$

in which form it is clear that the two leading factors will disappear upon taking the absolute square. The integration remaining is still daunting, but can actually be performed analytically in terms of the complex error function `Erf(x)`, allowing a reasonably efficient evaluation of the model's predictions, especially in the context of a symbolic-computation program such as Mathematica.

One notable success of this model is that it can produce predictions for the data obtained with one slit blocked, merely by performing the integration only over the region S_1 (or only over region S_2). Since the model makes no assumption about small angles, or detector at infinity, it gives predictions not limited by the Fraunhofer assumptions, which more successfully match the single-slit diffraction data readily taken with this apparatus. Nevertheless, this more sophisticated model does not change any qualitative conclusions you have established, and it makes only the slightest quantitative differences to your predictions; the need for deep thought about duality for light remains just as urgent after the models for its behavior have been improved.

It is worth noting that this Feynman model is silent about what the fundamental exponential varying in space really corresponds to. There is not even an appeal to the electromagnetic fields supposed to represent light; instead, the foundations of this model lie in quantum field theory, and the spatial variations represent mathematically the variation of these quantum fields. What is actually propagating through the air in your apparatus is a question not immediately addressed by the mathematics of the theory; it might be described as the propagation of a quantum amplitude, whose interpretation is that its absolute square gives the probability of a quantum event. However vague this picture might be physically, it at least has the virtue of showing that something can propagate as a wave disturbance, yet nevertheless predict the probability of observing a photon event. This is perhaps as close to an 'explanation' of wave-particle duality as can be achieved by a translation from the mathematical language of quantum field theory into a more mechanical or visualizable picture.

V. Conclusions

You have performed the two-slit experiment in three ways, and are now fully familiar with the apparatus required to perform it, the data that emerge, and some models that explain the data. You might even have some numerical conclusions, or deduced values for experimental parameters. But apart from this, you have some added insight into light's propagation and detection.

What is the best evidence your experiment provides that light has a wave-like nature?

What is the best evidence your experiment provides that light has a particle-like nature?

What is the concept of duality, and why was it invented?

Does duality satisfy you, or is the behavior of light paradoxical? Or, is it the character of your *description* of light that is paradoxical?

Appendix A. How to Read a Micrometer Drive

Two micrometer screws allow precise mechanical adjustments to the positions of the slit-blocker and the detector slit in this apparatus, and you should learn how they work, and how to read their scales. The two micrometers, and the mechanical flexure mounts they drive, are identical.

Each micrometer consists of a very carefully made metric screw thread of pitch exactly 0.50 mm, so that the rotating shaft of the micrometer advances 0.50 mm for each full (clockwise) turn of the screw. The markings on the micrometer are in place so you can

- keep track of the number of turns you've made, and thereby read the position of the shaft's working end to the nearest 0.50 mm; and
- interpolate within a full turn, so that you can finally quote the position of the shaft's working end to the nearest 0.01 mm .

Here's how to read the number of turns. Call the fixed part of the micrometer the 'barrel', and the rotating external part the 'drum'. Note that on the barrel there is a printed longitudinal 'stem', along which there are 'branches' emerging alternately on either side. On the one side, every fifth mark is labeled with an integer, 0, 5, 10, and so on: these are at 5-mm spacing, and between them are the 1-mm marks. On the other side of the stem are also branches at 1-mm spacing, but these lie halfway between the mm-marks, and form the 'half-mm' marks. Now turn the drum until the 0-mark on its circumference lies right along the line of the stem (this marks the 0° point on one of its 360° rotations). Find the last branch exposed to view on the barrel, and use it to read the micrometer to the nearest 0.50 mm.

For example, if the 5-branch and the next branch on the other side of the stem are the last exposed to view, the micrometer is set to 5.50 mm.

Now from that position, further counter-clockwise rotation of the drum will withdraw the screw, by another 0.50 mm for a full turn. If you rotate by a fraction N/50 of a full turn, you'll withdraw the micrometer by (N/50) x 0.50 mm, or (0.01 x N) mm. The drum's periphery is conveniently printed with 50 marks around the circumference, and every fifth one of these is labeled, so that you can read the integer N directly. This provides the rest of the information you need to read the micrometer to 0.01 mm.

Appendix B. Quantitative Detection by Lock-In Techniques

This appendix describes an alternative method for processing the photodiode signal which you used quantitatively in section III.B. of this manual; at the cost of a bit of electronic complexity, it offers higher sensitivity to the laser-derived light signal, and immunity from certain kinds of light induced and electronic background signals. The idea is to isolate that part of the signal due to the laser alone, and to discriminate against any other signals, by turning the laser on and off repeatedly, and electronically isolating the difference that this makes. This technique requires an oscillator with TTL-compatible output to modulate the laser, and a lock-in amplifier to process the photodiode signal.

To use this technique, note that the LASER MOD. input on the light-source end of your apparatus accepts a TTL-level input (0 to +4 Volt, square-edged waveform), which turns the laser *on* for TTL level 'high' (or connector left open) and turns the laser *off* for TTL level 'low' (or connector grounded). You can use any signal generator with TTL-compatible square-wave output to drive this modulation input; you will need modulation rates only in the 40 - 400 Hz range.

At the detector end of the apparatus, you will note there is the photodiode-amplifier output connector, which produces a voltage signal proportional to the optical power incident on the photodiode. (In addition there may be a small DC offset in this signal.) The photodiode's intrinsic response time is well under 1 ms, and the amplifier which turns it sub-μA output current into the voltage you've been using has a time constant of 0.4 ms.

If you do modulate the laser source, then this photodiode-voltage output will also be modulated in an approximate square wave. You can use lock-in detection of this output voltage, synchronously triggering the lock-in amplifier with the same TTL waveform which is modulating the laser, and thereby achieve total immunity from the DC offset of the photodiode amplifier, and nearly total immunity from all noise signals in the photodiode output. This will permit operation of the apparatus in the quantitative mode of section II.B. with the cover open, and/or higher-sensitivity detection of weaker signals on the photodiode.

Because of the 0.4-ms time constant in the photodiode voltage signal, you should use a laser modulation frequency below 400 Hz in this lock-in mode. Some lock-in amplifiers have a current-mode input, in which case you can send the raw photodiode current directly to the lock-in; this permits somewhat faster modulation. In either case, you'll want to set the lock-in's time constant to at least 100 ms, so as to average over many on-off modulation cycles of the laser.

Note that the lock-in technique will do nothing to discriminate against scattered *laser* light that might be reaching the photodiode; nor will it compensate for poor alignment of the apparatus. But given the averaging features and sensitivity of a typical lock-in amplifier, it can markedly improve the signal-to-noise ratio and the dynamic range of your measurements.

Appendix C. Injecting Test Pulses into the PMT Amplifier

This appendix describes a way to use artificial stimulation of the pulse amplifier and discriminator in the detection box at the end of the apparatus; it requires an electronic generator and oscilloscope, and offers the chance to understand and calibrate the pulse-processing electronics in a context separate from actual photon events.

In normal operation, the photomultiplier tube turns single-photoelectron events at the photocathode into much-amplified pulses of electrons arriving at the anode; depending on the high-voltage setting of the PMT supply, the gain of the PMT's dynode structure might be 10^5 or more. A charge pulse of this size is sufficient to excite the charge-pulse amplifier, whose output is in turn sufficient to trigger the TTL-output pulse generator in the detection electronics. The goal here is to test these electronics with a source of artificial events, which are more frequent and regular than those due to individually and randomly arriving photons.

A signal generator will stimulate the artificial events; you will need a square-wave generator capable of delivering sharp-edged transitions (<20 ns transition time) of amplitude smoothly adjustable down to say 25 mV. You'll also need a coaxial attenuator of 50-Ω impedance and 20 dB (or more) attenuation, and an adaptor to deliver its output to the SMA-type input connection marked INPUT TEST PULSE on the detector box.

To conduct these tests, ensure that the PMT high-voltage supply is set to 0 V, so that no light-induced pulses will be reaching the amplifier input. Now connect the square-wave generator's output to a BNC cable to bring it to a tee-junction at an ocilloscope's input; use a high impedance input on the 'scope to monitor the voltage swings ΔV of the square wave as it goes by. Send the square waveform onward from the tee by another BNC cable to the coaxial attenuator, and couple that attenuator's output to the SMA test-input connector.

Now suppose you adjust the square-wave generator amplitude to give voltage swings visible at the 'scope of ± 40 mV. Since a 20-dB attenuator reduces power by a factor of 100, in a system of 50-Ω impedance it attenuates both voltage and current by a factor of 10. Thus you can be confident that clean fast transitions of $\Delta V = \pm 4.0$ mV are present at the test-input connector.

These transitions are internally terminated by a 50-Ω resistor, and then coupled by a 2-pF capacitor to the input of the pulse amplifier. The negative-going voltage edge will thus couple into the amplifier a charge pulse of size $q = C\,\Delta V = (2 \times 10^{-12}\text{ F})\,(-4 \times 10^{-3}\text{ V}) = -8 \times 10^{-15}$ C, which is the charge carried by 5×10^4 electrons. This charge pulse will be your surrogate for the pulse of electrons arriving at the anode of the PMT, and it will excite the pulse amplifier and discriminator in exactly the same way as genuine photon events do. The advantage of using the square-wave generator to stimulate these events is that the test pulses can be frequent (say at rate 10 kHz), regularly spaced (say every 100 μs), and accompanied by a synchronizing pulse (to trigger a 'scope or other equipment).

With such excitation of the input of the pulse amplifier, monitor with a 'scope the analog pulse at the PHOTOMULTIPLER OUT BNC connector. You'll be able to see just what response it gives to a fast charge pulse of a quantifiable size; alternatively, you'll be able to say just how much charge must be delivered in a fast pulse to its input in order that its output pulse reaches an amplitude of (say) +50 mV. That, in turn, will tell you how high the gain of the dynode structure of the PMT needs to be, for single-photoelectron events at the photocathode to be able to give 50-mV analog pulses at this monitor output.

Furthermore, with this steady train of (say) 50-mV pulses at the analog output, you will be able to find just what discriminator setting is required for them to trigger reliably the discriminator, and thus produce a train of TTL-level pulses at its OUTPUT TTL BNC connector.

Finally, with all these electronic tests completed, understood, and documented, you can remove the artificial test-pulse excitation, and be assured that the whole train of electronics will respond in the same way to actual electron pulses from the PMT anode as it has to your test pulses.