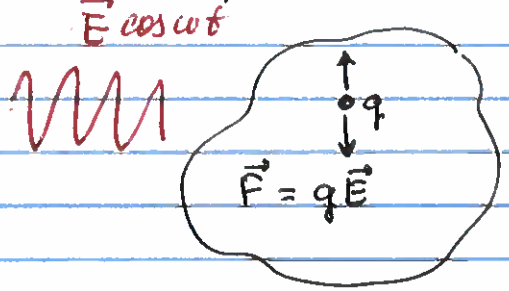
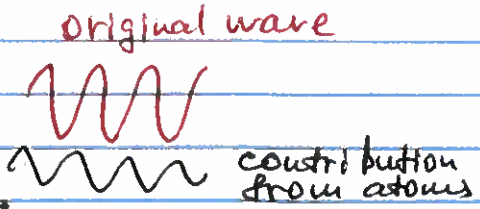
Light–atom interactions: refraction, absorption, scattering

Our simple model of interaction

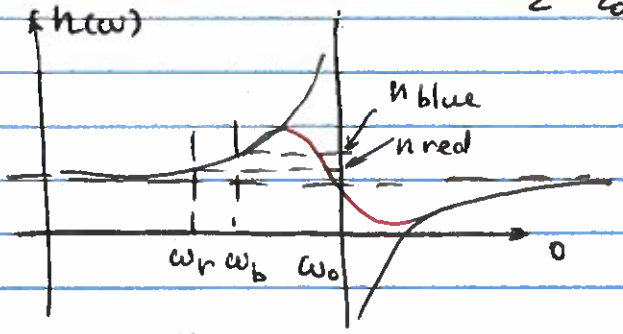




$$\vec{p} = N \cdot \vec{d} = N \cdot q \cdot x \cos(\omega t + \varphi)$$

Last time we "calculated" refractive index $n(\omega)$ to be

$$n(\omega) = 1 + \frac{1}{2}\frac{Ne^2}{\varepsilon_0 m_e}\frac{1}{\omega_0^2 - \omega^2} = 1 + \frac{\omega_p^2}{2}\frac{1}{\omega_0^2 - \omega^2}$$



here ω – freq. of EM field
ω_0 – natural atomic resonance frequency

Our simple model is quite accurate when we are not near the atomic resonant frequency.

Why the refractive index for glass (or water) is larger for blue light than for red light? $\omega_{red} < \omega_{blue}$

Because these materials have resonances in UV part of the spectrum

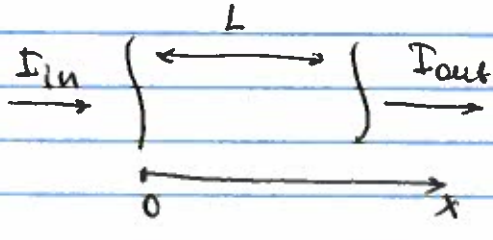
Our model fails near the resonance, because we didn't include any losses, and "allow" e-m field to put ~~as much~~ ~~error~~ potentially unlimited power into atomic oscillator. This in unphysical. In fact, any oscillator has its decay rate γ – the inverse of the time it takes for the oscillation to ~~not~~ decay (attenuate) with no driving force present.
For atoms γ characterize how quickly the atomic dipole will disappear

More accurate expressions

Refractive index $n(\omega) \approx 1 + \frac{\omega_p^2}{4\omega} \frac{\omega_0 - \omega}{(\omega_0 - \omega)^2 + \gamma^2/4}$

Because we added loss of energy channel, we also enable light absorption

Light attenuation



~~I_{out}~~ $I_{out} = I_{in} e^{-\beta \cdot L}$

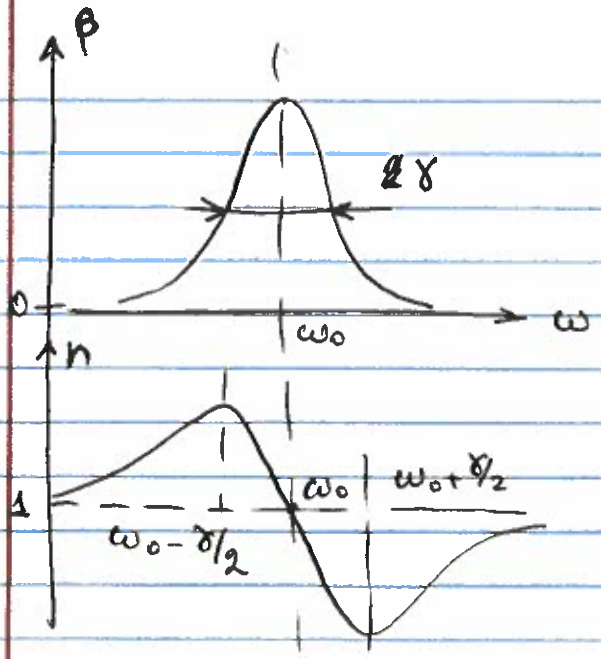
in general $I(x) = I(0) e^{-\beta \cdot x}$

β – absorption coefficient

In our resonant material

$$\beta(\omega) = \beta_0 \frac{\gamma}{(\omega_0 - \omega)^2 + \gamma^2/4} \qquad \beta_0 = \frac{\omega_p^2}{2c\gamma}$$

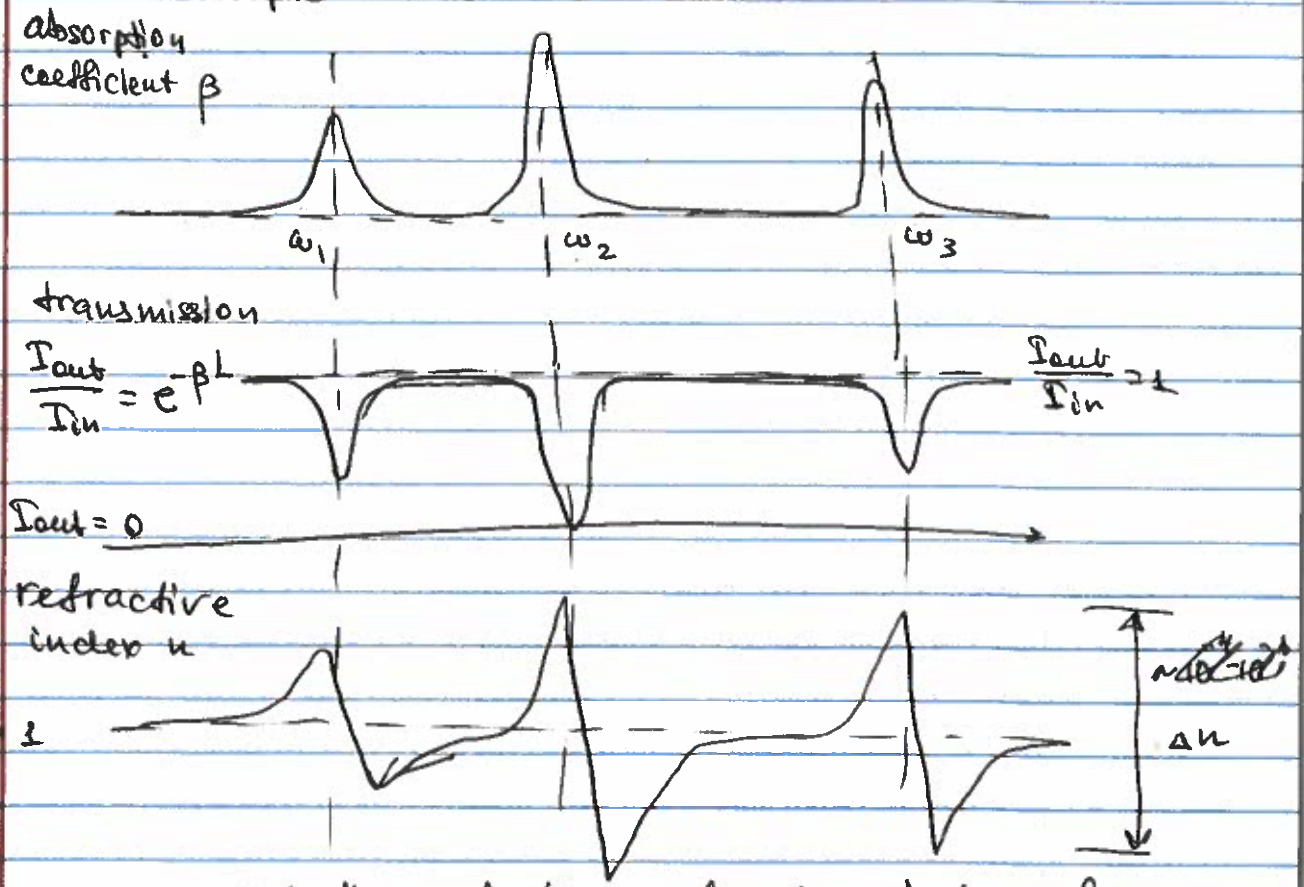
resonant absorption



resonant absorption strongest around ω_0
resonance linewidth = 2γ

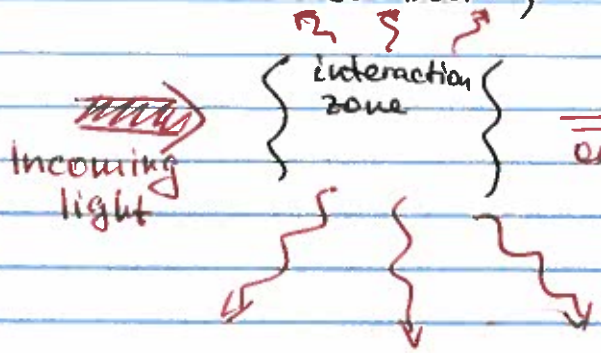
matching variation in the refractive index (dispersive shape)

If an atom / molecule / structure has multiple internal resonances



variation of the refractive index Δn for gasses is very small $\Delta n \sim 10^{-4} - 10^{-6}$
since gases have much lower density

If the light (energy) from our original wave is absorbed, where does it go?



light can be reemitted in all directions
- fluorescence (individual atoms/molecules)
- scattering (often larger particles, but sometimes molecules)

Classical dipole emission

Original EM field $\vec{E}_0 \cos\omega t$ → induced dipole $\vec{d} = \alpha \varepsilon_0 \vec{E}_0 \cos\omega t$

α - polarizability (represent "stiffness of the oscillator")

Total power emitted by the dipole

$$W_{dipole} = \frac{\omega^4 \alpha^2 \varepsilon_0 E_0^2}{12\pi c^3} = \frac{\omega^4}{c^4} \frac{\alpha^2}{6\pi} W = \frac{8\pi}{3} \frac{\alpha^2}{\lambda^4} W$$

Power of the incoming wave $W = \frac{1}{2} \varepsilon_0 c E_0^2$

Power lost from the wave due to dipole radiation

$$dW = -N dz \cdot W_{dipole} = -\frac{8\pi^3 \alpha^2 N}{3\lambda^4} W$$

$$-\frac{1}{W}\frac{dW}{dz} = \beta = \frac{8\pi^3 \alpha^2 N}{3\lambda^4}$$

For classical dipole emission loss coefficient $\beta = \frac{8\pi^3 d^2 N}{\lambda^4} = \frac{32}{3}\pi^3 \frac{(n-1)^2}{N\lambda^4}$

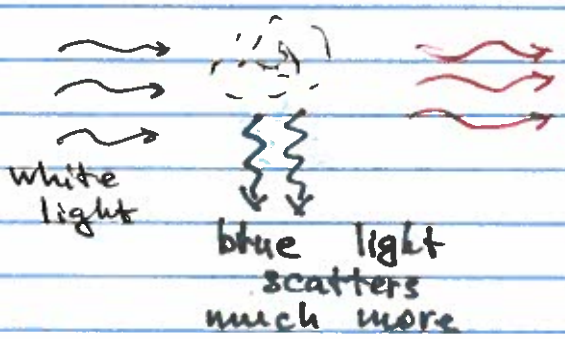
This model does not work for atoms and molecules, since their decay is governed by very different rules of quantum mechanics.
But it works for very small particles (size $\ll \lambda$)

Rayleigh Scattering
very strong wavelength dependense

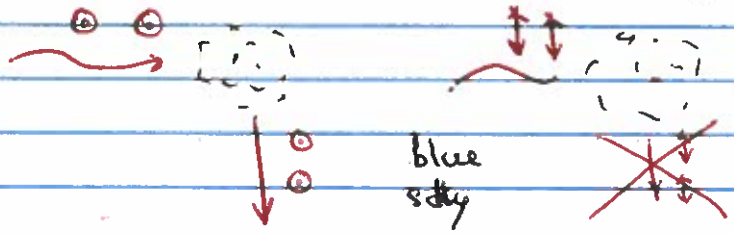
$\lambda_{red} \approx 2\,\lambda_{blue}$

$\frac{1}{\lambda_{red}^4} = \frac{1}{16}\,\frac{1}{\lambda_{blue}^4}$

red scatters much less!



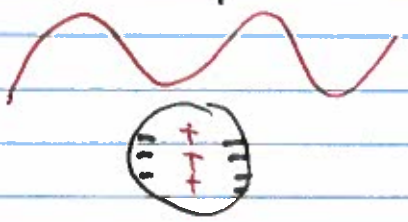
Moreover, Rayleigh scattering is polarizing



red sunset

Try seeing it with a polarizer on a clear day

Larger particles (size ~ wavelength)
Mie scattering



Much more complex problem, since EM wave creates charge distribution inside the particle

Much weaker wavelength dependence
white light scatters white
(white clouds)