Optical systems (combination of lenses)

In general, however, it is possible to include mirrors, transparent flats, etc.

1. Graphical method

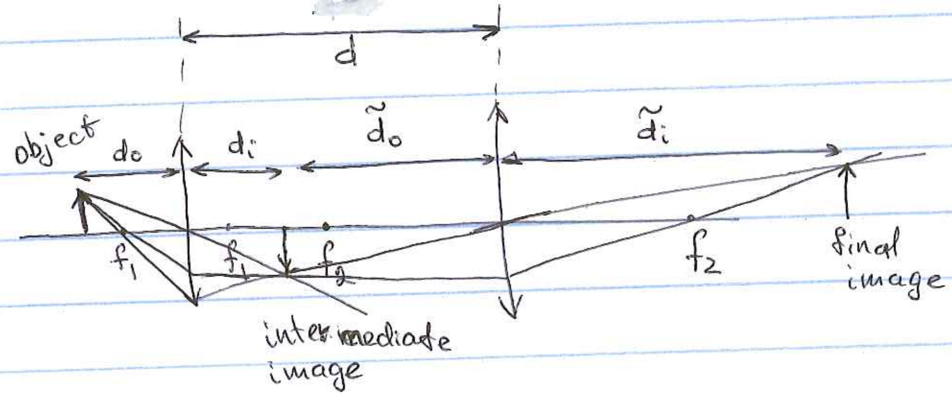
   Find an image of the first lens, it becomes the object for the consecutive lens

2. Mathematical method

   Find the position of an intermediate image, repeat for consecutive lenses

3. Matrix method (brief overview)

Two positive lenses



First lens: $\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f_1}$ $\qquad d_i = \frac{d_o \cdot f_1}{d_o - f_1}$

$\tilde{d}_o = d - d_i$

Second lens $\frac{1}{\tilde{d}_o} + \frac{1}{\tilde{d}_i} = \frac{1}{f_2}$ $\qquad \tilde{d}_i = \frac{\tilde{d}_o \cdot f_2}{\tilde{d}_o - f_2}$

$$\tilde{d}_i = \frac{f_2 \left[d(d_o - f_1) - f_1 d_o \right]}{(d - f_2)(d_o - f_1) - f_1 d_o}$$

Magnification of the system of the lenses is equal to the product of magnifications for each individual lens

Hight of the intermediate image $h_i = M_1 h_o$

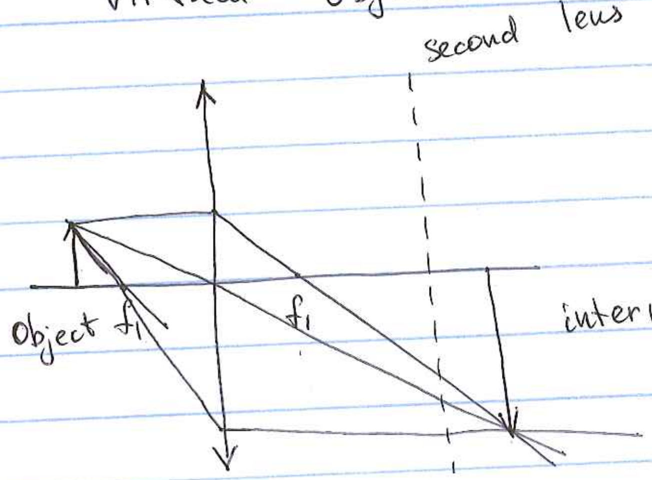
— " — second — " — $\tilde{h}_i = M_2 h_i = \underbrace{M_2 M_1}_{\text{total magnification}} h_o$

$$M_1 = -\frac{f_1}{d_o - f_1} \qquad M_2 = -\frac{f_2}{\tilde{d}_o - f_2} = -\frac{f_2}{d - \frac{d_o f_1}{d_o - f_1} - f_2}$$

$$M = M_1 \cdot M_2 = \frac{f_1 f_2}{(d - f_2)(d_o - f_1) - d_o f_1}$$

These calculations are valid for any value of f_1, f_2 and d.

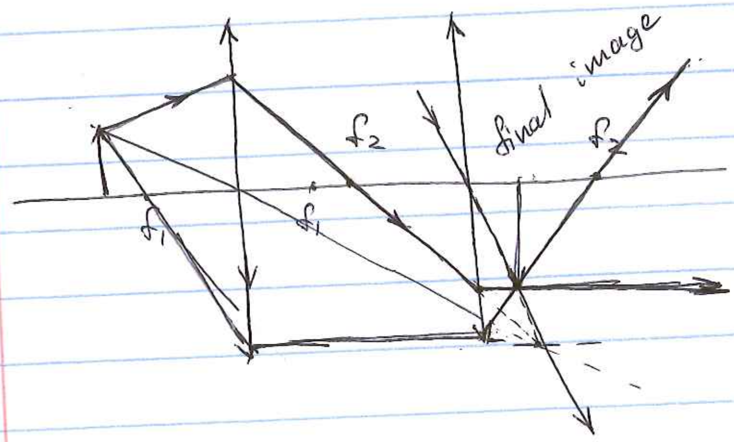
Virtual object



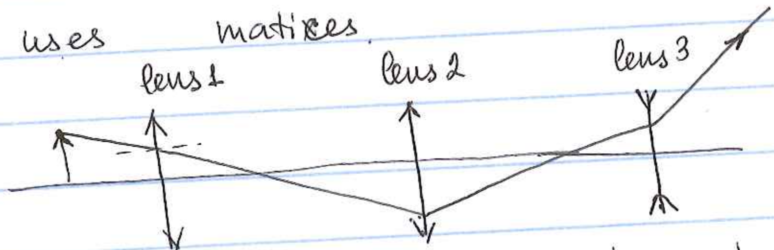
intermediate image

Since $d_i > d$, $\tilde{d}_o = d - d_i < 0$

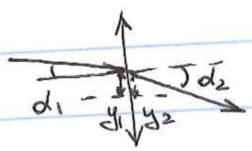
— virtual object for the second lens



For several lenses we can do such calculations, even though they are rather tedious.
General method of optical system calculations uses matrices.



If I know how the beam's angle changes at each lens, at at what point of the lens it hits, I can predict how the beam travels.



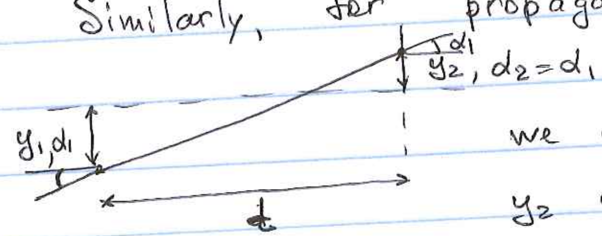
Immediately before and after lens the beam changes direction, but not its height

$y_1 = y_2$, $\alpha_1 \to \alpha_2$ If we know a rule how lens changes the angle, we can calculate it for any beam

$\boxed{y_1, \alpha_1} \Rightarrow \boxed{\text{lens}} \Rightarrow \boxed{y_2, \alpha_2}$

input parameters S_L output parameters

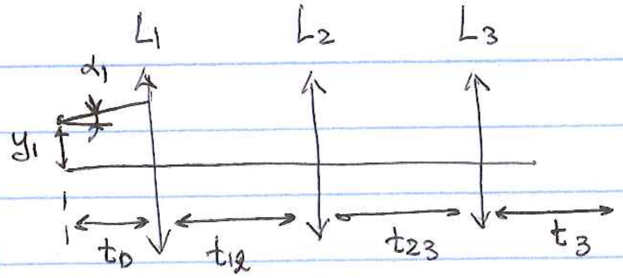
Similarly, for propagation



we can figure out how y_2 changes, if we know y_1, α_1

$\boxed{y_1, \alpha_1} \Rightarrow \boxed{\text{propagation distance } d} \Rightarrow \boxed{y_2, \alpha_2}$

S_T



$\boxed{y_1, \alpha_1} \Rightarrow S_{t_0} \Rightarrow \boxed{y_2 \alpha_2} \Rightarrow S_{L_1} \Rightarrow \boxed{y_3, \alpha_3} \Rightarrow S_{t_{12}} \Rightarrow \ldots \Rightarrow S_{t_3} \Rightarrow \begin{matrix} y_f \\ \alpha_f \end{matrix}$

S_L and S_t can be represented as 2×2 matrices

$\begin{pmatrix} \alpha_1 \\ y_1 \end{pmatrix}$ – initial beam parameters

Translation matrix $S_t = \begin{pmatrix} 1 & 0 \\ t & 1 \end{pmatrix}$

$$\begin{pmatrix} \alpha_2 \\ y_2 \end{pmatrix} = S_t \begin{pmatrix} \alpha_1 \\ y_1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ t & 1 \end{pmatrix} \begin{pmatrix} \alpha_1 \\ y_1 \end{pmatrix} = \begin{pmatrix} \alpha_1 \\ y_1 + \alpha_1 \cdot t \end{pmatrix}$$

Lens matrix $S_L = \begin{pmatrix} 1 & -1/f \\ 0 & 1 \end{pmatrix}$

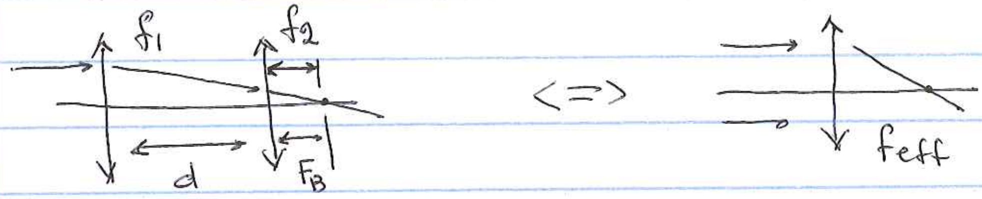
$$\begin{pmatrix} \alpha_3 \\ y_3 \end{pmatrix} = S_L \begin{pmatrix} \alpha_2 \\ y_2 \end{pmatrix} = \underbrace{S_L \cdot S_t}_{\text{we know}} \begin{pmatrix} \alpha_1 \\ y_1 \end{pmatrix}$$

we know how to multiply matrices

$$\begin{pmatrix} \alpha_f \\ y_f \end{pmatrix} = S_{t_3} S_{L_3} S_{t_{23}} S_{L_2} S_{t_{12}} S_{L_1} S_{t_0} \begin{pmatrix} \alpha_1 \\ y_1 \end{pmatrix}$$

Different matrices can be used to describe different optical elements to trace optical beams in any complicated system.

Using the matrix method, one can calculate the effective focal length f_{eff} of two thin lenses



$$\frac{1}{f_{eff}} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{d}{f_1 f_2}$$

Back focus $\quad F_B = \left(1 - \frac{d}{f_1}\right) \cdot f_{eff}$