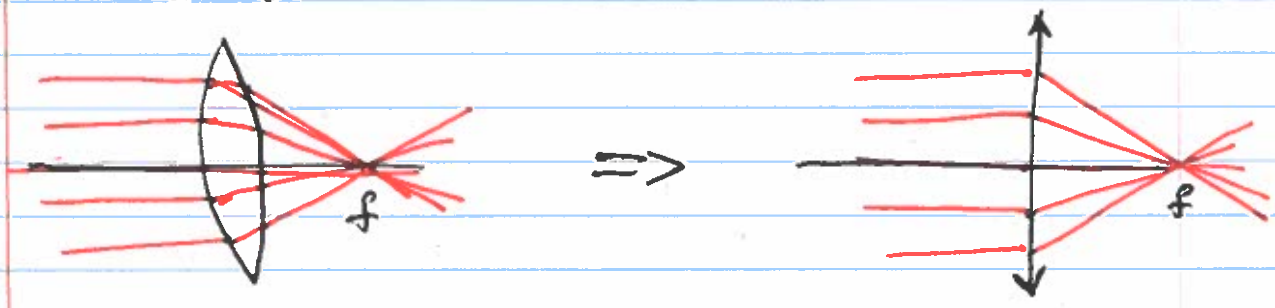
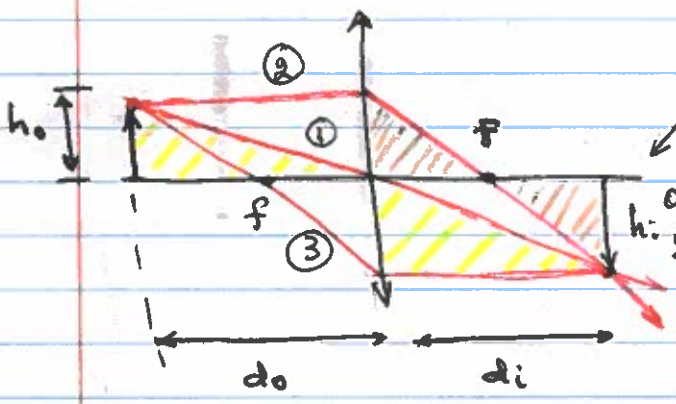
Image formation with a convergent lens

For many simple imaging designs we simplify all the refraction steps inside the lens, and use "thin lens approximation"

Instead of



How to construct the image by ray tracing



real inverted image

expect $|m| > 1$ (enlarged)

$m < 0$ (inverted)

Two sets of similar triangles (yellow) $\frac{h_i}{h_o} = \frac{d_i}{d_o}$

$$\frac{h_i}{h_o} = \frac{d_i - f}{f}$$

$$\frac{d_i - f}{f} = \frac{d_i}{d_o} \Rightarrow \frac{d_i}{f} - 1 = \frac{d_i}{d_o} \Rightarrow \frac{1}{f} - \frac{1}{d_i} = \frac{1}{d_o}$$

Lens equation

$$\boxed{\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f}}$$

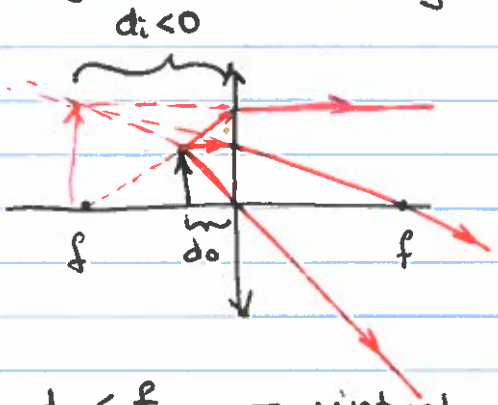
$$m = -\frac{h_i}{h_o}$$

Real image / object → ~~d~~ $d_i, d_o > 0$

Virtual image / object $d_i, d_o < 0$

To know exactly where the image is we need to do the calculations, but we often can predict the general properties of an ~~image~~ object based on where it is with respect to the focus

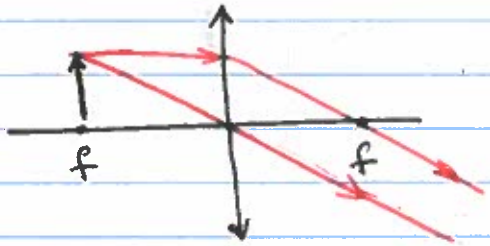
Object is very close to the lens ($d_o < f$)



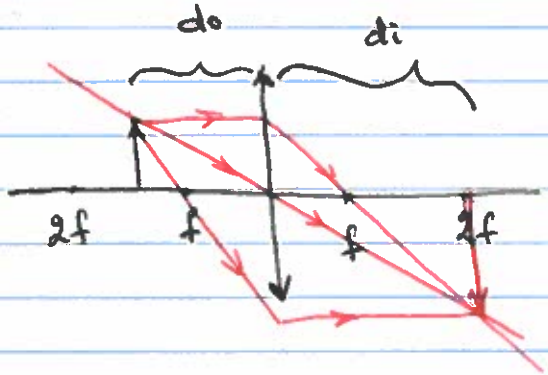
no real image
virtual image
located at a position
from which
the refracted beams
seem to originate

$d_o < f$ – virtual, erect, magnified image

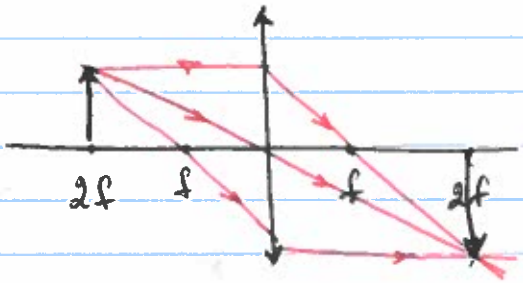
$d_o = f$ – no image



$f < d_o < 2f$ – real, inverted image
magnified image



$d_o = 2f$



$d_i = d_o = 2f$

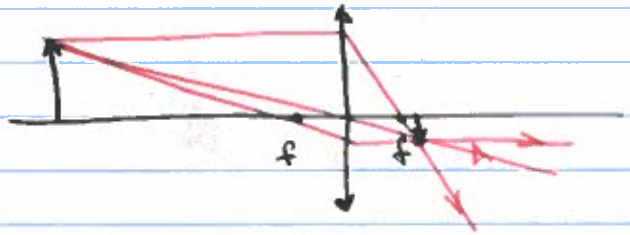
$m = -\frac{d_i}{d_o} = -1$

real, inverted image same size

Sometimes this configuration is called 4f-imaging, since this the arrangement that gives you the true size of the object

$d_o > 2f$

real, inverted, reduced image



Three main imaging rays:

1. Goes through the center of the lens → no change in direction
2. Goes parallel to the optical axis → crosses focus after the lens
3. Goes through the focus before the lens → → travels parallel to the axis after the lens

Same results, using lens equation

$$\frac{1}{d_i} + \frac{1}{d_o} = \frac{1}{f} \qquad \frac{1}{d_i} = \frac{1}{f} + \frac{1}{d_o} = \frac{d_o - f}{d_o \cdot f}$$

$$\boxed{d_i = \frac{d_o \cdot f}{d_o - f}}$$

We assume real object.

$d_o > 0$

if $d_o > f \rightarrow d_i > 0$ real image

$d_o < f \rightarrow d_i < 0$ ~~inverted~~ virtual image

Magnification $\quad m = -\frac{d_i}{d_o} = -\frac{f}{d_o - f}$

if $d_o > f \rightarrow d_i > 0 \rightarrow m < 0$ inverted image

if $d_o < f \rightarrow d_i < 0 \rightarrow m > 0$ erect image

if $d_o > 2f \rightarrow d_o - f > f \quad |m| = \frac{f}{d_o - f} < 1$

if $f < d_o < 2f \rightarrow 0 < d_o - f < f \quad |m| = \frac{f}{d_o - f} > 1$

"Special" cases $\quad d_o = 2f \quad d_i = \frac{d_o \cdot f}{d_o - f} = \frac{2f^2}{f} = 2f$

$m = -\frac{d_i}{d_o} = -1 \quad |m| = 1$

$d_o = f \quad d_o - f = 0 \quad d_i = \frac{d_o \cdot f}{d_o - f} \rightarrow \infty$