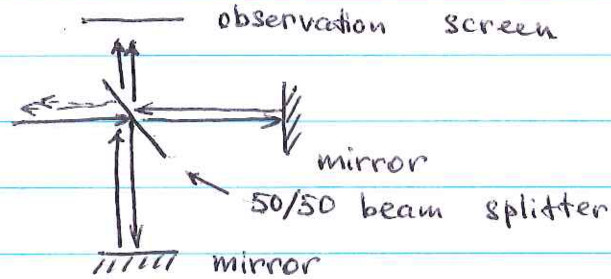
Interferometers

The general principle of the interferometer: divide the (laser) beam in two, let them travel two different paths, recombine and observe the interference

Michelson interferometer



If one of the mirrors moves by $\lambda/2$, the interference switches from constuctive to destructive

The most famous Michelson interferometer LIGO

Physical length of each arm - 4 km

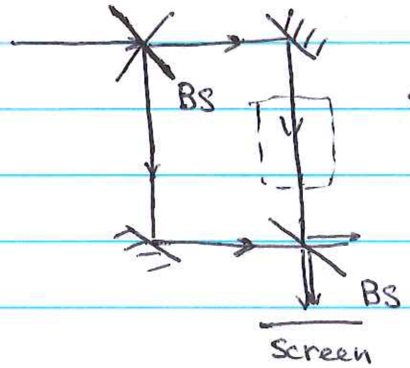
Effective length - 1000 km (because of the extra bouncing from intermediate mirrors)

$$\frac{\Delta L}{L} \approx \frac{\lambda}{L} \sim \frac{10^{-6}\,m}{10^{7}\,m} = 10^{-13}$$

This relative change is just too obvious (max to min).

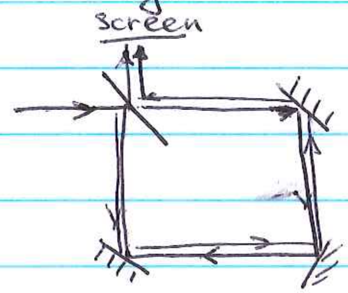
Real LIGO $\Delta L/L \sim 10^{-23}$

Mach-Zender interferometer



single pass through a test volume

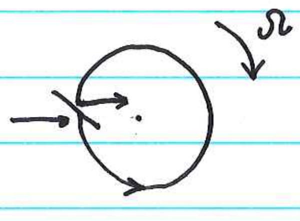
Sagnac interferometer



also known as ring cavity

Sagnac interferometer is used for laser gyroscopes

(To simplify geometrical calculations, let's assume light travels in circles)



time of one roundtrip

$$T = \frac{2\pi R}{c}$$

during this time the mirror moves

$$\Delta\Theta = \Omega T = \Omega \frac{2\pi R}{c}$$

Assuming that mirror moves much slower than light

$\Delta x = R\Delta\Theta = 2\underbrace{\pi R^2}\Omega/c = 2\underbrace{A}\frac{\Omega}{c}$ ~~phase shift~~ $\cancel{\Delta\varphi = \frac{2\pi}{\lambda}\Delta x}$

~~The beams moving in an opposite direction~~

Since in our example the beam moves at the same direction as the mirror, it has to travel a bit longer each time to complete the round trip, giving it an extra phase shift

$$\Delta\varphi_+ = \frac{2\pi}{\lambda}\Delta x = \frac{4\pi}{\lambda} A\,\Omega/c$$

However, a beam travelling in an opposite direction will meet the mirror sooner, as it moves toward it, reducing its ~~pas~~ round-trip by Δx, and thus its phase is $\Delta\varphi_- = -\frac{2\pi}{\lambda}\Delta x = -\frac{4\pi}{\lambda} A\,\Omega/c$, compare to a stationary case

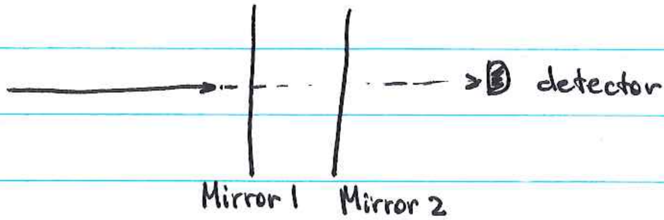
Phase difference b/w two beams

$$\Delta\varphi = \Delta\varphi_+ - \Delta\varphi_- = \frac{8\pi}{\lambda} A\,\Omega/c$$

The phase difference is proportional to the rotation rate (and the area enclosed by the interferometer)

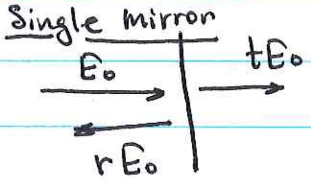
Multi-pass interferometers (cavities)

Fabri - Perot interferometer



Light can bounce between the two mirrors and interfere with itself

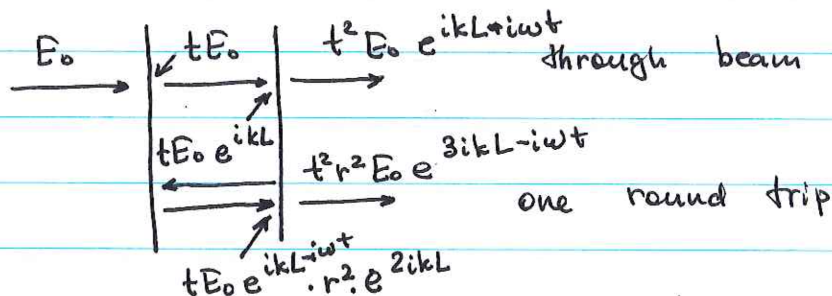
Single mirror



t, r – transmission and reflection coefficients

For a good mirror $r \sim 1$, $t \ll 1$

Two mirrors



$$E_{tot} = \mathrm{Re}\{t^2 E_0 e^{ikL-i\omega t} + t^2 r^2 E_0 e^{3ikL-i\omega t} + t^2 r^4 E_0 e^{5ikL-i\omega t} + \dots\} =$$

$$= \mathrm{Re}\{t^2 E_0 e^{ikL} \underbrace{(1 + r^2 e^{2ikL} + r^4 e^{4ikL} + \dots)}_{\text{interference between multy pass beams}}\}$$

$$= \mathrm{Re}\left\{\frac{t^2 E_0 e^{ikL-i\omega t}}{1 - r^2 e^{2ikL}}\right\} = \frac{t^2 E_0 \cos(kL-\omega t)}{1 - r^2 \cos 2kL}$$

$$I_{tot} = \langle E_{tot}^2 \rangle_{time} = \frac{1}{2} E_0^2 \frac{t^2}{1 - r^2 \cos 2kL} = I_0 \frac{t^2}{1 - r^2 \cos 2kL}$$

Resonance conditions (all multipass beams interfere constructively) : $\cos 2kL = 1$ $\quad \cancel{2}kL = \cancel{2}\pi m$

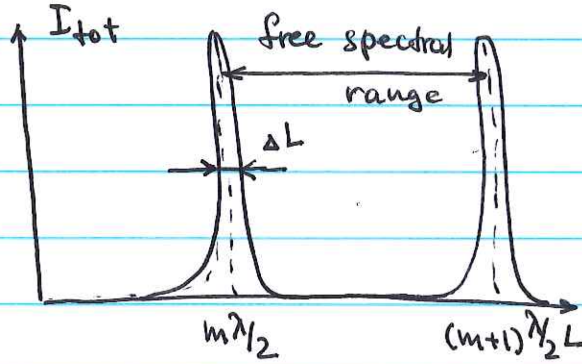
$$\frac{2\pi L}{\lambda} = \pi m$$

$$L = m \cdot \frac{\lambda}{2}$$

Under these conditions each consecutive reflection is shifted in phase by integer # of wavelength.

Then $\quad I_{tot} = I_0 \dfrac{t^2}{1-r^2} = I_0 \quad (\text{if } t^2 + r^2 = 1)$

all light is transmitted through a pair of very reflecting mirrors!



$$\Delta L = \frac{\lambda}{4\pi}\,\frac{1-r^2}{r}$$

The higher is R, the sharper are the transmission resonances

$$\text{Finesse} = \frac{\text{width of the peak}}{\text{separation b/w two peaks}} = \pi \frac{r}{1-r^2} = \pi \frac{\sqrt{R}}{1-R}$$

(r is the ~~transmission~~ reflection coefficient for the amplitude, and $R = r^2$ – for intensity)

Finesse gives an estimate on the number of the round trips a photon can make inside the cavity before escaping (or being lost).