Ray optics vs wave optics

To describe a ray, one needs to specify a propagation direction and position, color (for a spectroscopically pure light), and intensity (i.e. energy per area and time)

$$\text{Intensity} = \frac{\text{energy}}{\text{area} \cdot \text{time}}$$

To describe a wave, we need a lot of same things! Propagation direction* $(\vec{k})$, frequency (ω or λ), and amplitude

$$\vec{E}(\vec{r},t) = \vec{E}_0 \cos(\vec{k}\cdot\vec{r} - \omega t + \varphi)$$

* to avoid complications with vectors, we usually direct the z axis along the propagation direction, so that $\vec{k}\cdot\vec{r} \equiv kz$

Main difference:

For rays we add intensities

For waves we add ~~amplitudes~~. field strengths (superposition principle)

$$\vec{E}_{total} = \vec{E}_1 + \vec{E}_2 + \vec{E}_3 + \dots$$

Intensity $= \langle |\text{Amplitude}|^2 \rangle_{\text{time average}}$

Imagine we have two identical light sources, such that their radiation ovelaps at some point in space, and

$$\left.\begin{aligned} \vec{E}_1 &= \vec{E}_0 \cos(kz - \omega t) \\ \vec{E}_2 &= \vec{E}_0 \cos(kz - \omega t + \varphi) \end{aligned}\right\}$$

same frequency, same polarization same amplitude

Intensities: $I_1 = \langle |E_1|^2 \rangle = E_0^2 \langle \cos^2(kz-\omega t) \rangle = \frac{1}{2} E_0^2$

$I_2 = \langle |E_2|^2 \rangle = E_0^2 \langle \cos^2(kz-\omega t+\varphi) \rangle = \frac{1}{2} E_0^2$

$I_{total} = E_0^2 \leftarrow$ we expect that for ray treatment

Field strengths: $\vec{E}_{tot} = \vec{E}_0 \left(\cos(kz-\omega t) + \cos(kz-\omega t+\varphi) \right) =$

$$\boxed{\begin{aligned} &\text{Math. identity} \\ &\cos\alpha + \cos\beta = \\ &= 2\cos\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2} \end{aligned}}$$

$$= 2\vec{E}_0 \cos\left(kz-\omega t+\frac{\varphi}{2}\right)\cos\frac{\varphi}{2}$$

$$I_{tot} = \langle |\vec{E}_{tot}|^2 \rangle = 4 E_0^2 \cos^2\frac{\varphi}{2} \left\langle \cos^2\left(kz-\omega t+\frac{\varphi}{2}\right) \right\rangle =$$

$$\underline{I_{tot} = 2E_0^2 \cos^2 \varphi/2}$$

Intensity depends strongly on the relative phase of the two waves.

a) if $\varphi = 0$ or $\pm 2\pi$ or any multiples of 2π

[constructive interference] $I_{tot} = 2E_0^2 \longrightarrow$ 2 times higher than ~~for~~ expected for ray optics

b) if $\varphi = \pm\pi, \pm 3\pi$ or any odd number of π

$I_{tot} = 0$ [destructive interference]

Why it is hard to observe interference?

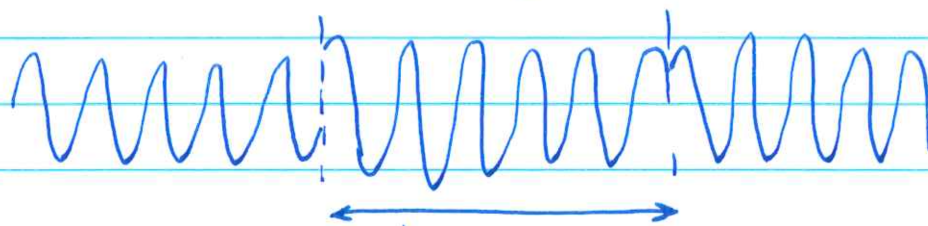
1. Two optical fields must have the same frequency

$$\cos(\omega_1 t + \varphi_1) + \cos(\omega_2 t + \varphi_2) = 2\cos\left(\frac{\omega_1+\omega_2}{2}t + \frac{\varphi_1+\varphi_2}{2}\right)\cos\left[\frac{\omega_1-\omega_2}{2}t + \frac{\varphi_1-\varphi_2}{2}\right]$$

Optical frequencies are $\sim 10^{14}$ Hz, and normal photodetector won't record intensity variation at frequencies above ~MHz (10^6 Hz). Human eye is much worse — we need ~ 1 Hz difference b/w two optical fields to being able to notice the time-varying interference.

Solution: to use the light originated from the same source, by dividing it in two or more beams.

2. Even the best laser light can be approximated by a continuous harmonic wave only for a limited time / length



coherence length
or
cohererence time

If one tries to interfere light waves separated by more than a coherence length (or time), no good phase stability will be possible $\longrightarrow$ no interference

Laser coherence length $\sim$ a few meters
Discharge lamp $\sim$ a ~~few~~ fraction of a mm
Light bulb $\sim$ nanometers

The better is the coherent properties of a light source, the easier it is to see interference effects.