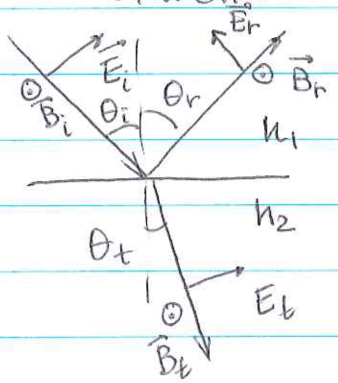
Laws of reflection and refraction

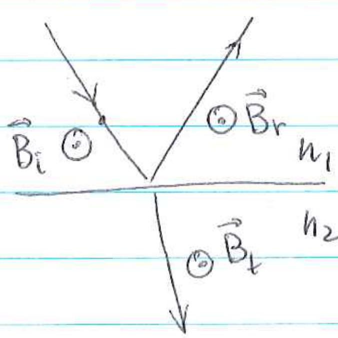
We know how the light beams reflect and refract – but we never were able to know how much is reflected and refracted! Maxwell's equs for the rescue!



$\vec{B}, \vec{E}, \vec{D}$ change values and directions on the boundary, but Maxwell's equations stay the same!

Boundary conditions

The component of total electric field and total magnetic field parallel to the boundary must be the same on both sides of the boundary

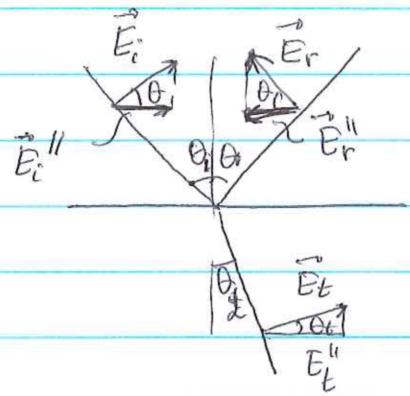


$$\vec{B}_i + \vec{B}_r = \vec{B}_t$$

$$B_i + B_r = B_t$$

$$E = v \cdot B = \frac{c}{n} B \Rightarrow B = \frac{n}{c} E$$

$$\boxed{n_1 E_i + n_1 E_r = n_2 E_t}$$



$$E_i^{\parallel} = E_i \cos\theta_1$$
$$E_r^{\parallel} = - E_r \cos\theta_1$$
$$E_t^{\parallel} = E_t \cos\theta_2$$

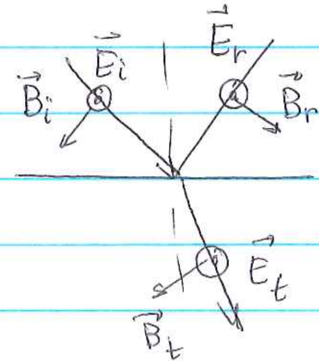
$$E_i^{\parallel} + E_r^{\parallel} = E_t^{\parallel}$$
$$\boxed{E_i \cos\theta_1 - E_r \cos\theta_1 = E_t \cos\theta_2}$$

Reflection coefficients (∥ means $\vec{E}$ is in the plane of insidence)

$$r_{\parallel} = \left(\frac{E_r}{E_i}\right)_{\parallel} = \frac{n_2\cos\theta_1 - n_1\cos\theta_2}{n_2\cos\theta_1 + n_1\cos\theta_2}$$

$$t_{\parallel} = \left(\frac{E_t}{E_i}\right)_{\parallel} = \frac{2n_1\cos\theta_1}{n_2\cos\theta_1 + n_1\cos\theta_2}$$

Another distinct configuration: $\vec{E}$ is perpendicular the insidence plane



$$E_i + E_r = E_t$$
$$B_i\cos\theta_1 - B_r\cos\theta_1 = B_t\cos\theta_2$$
$$n_1 E_i\cos\theta_1 - n_1 E_r\cos\theta_1 = n_2 E_t\cos\theta_2$$

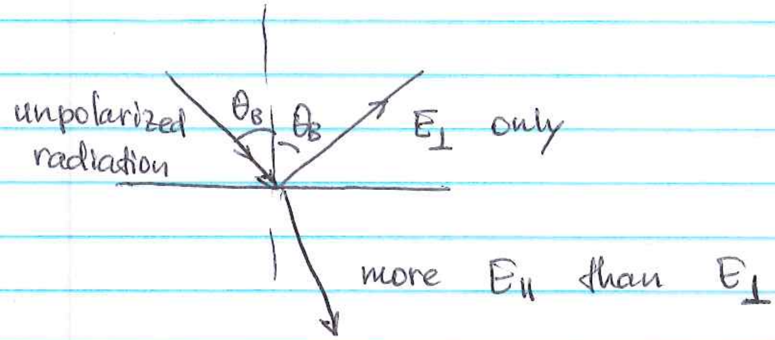
$$r_{\perp} = \left(\frac{E_r}{E_i}\right)_{\perp} = \frac{n_1\cos\theta_1 - n_2\cos\theta_2}{n_1\cos\theta_1 + n_2\cos\theta_2}$$

$$t_{\perp} = \left(\frac{E_t}{E_i}\right)_{\perp} = \frac{2n_1\cos\theta_1}{n_1\cos\theta_1 + n_2\cos\theta_2}$$

Intensity reflection

$$R = r^2 \qquad T = n\left(\frac{\cos\theta_2}{\cos\theta_1}\right)t^2$$

(or $T = 1 - R$) because of the energy conservation



This is the principle of polarizing sunglasses: sunlight reflected off the road is partially polarized, so if the sunglasses block this polarization, they reduse the effect of glares.

For normal incidence $\theta_1 = \theta_2 = 0$

$$r_\parallel = r_\perp = \frac{n_1 - n_2}{n_1 + n_2} \quad (<0, \ \pi\text{-phase shift if } n_1 < n_2)$$

$$R = \left(\frac{n_1 - n_2}{n_1 + n_2}\right)^2$$

The larger is the refractive index difference, the stronger is reflection

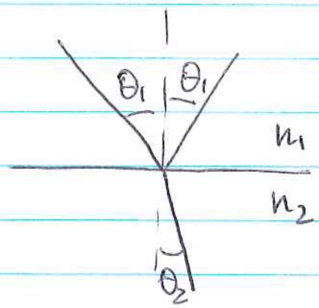
Water / air $\quad n_1 = 1 \quad n_2 = 1.33 \quad R = \left(\frac{0.33}{2.33}\right)^2 = 0.02$

Glass / air $\quad n_1 = 1 \quad n_2 = 1.5 \quad R = \left(\frac{0.5}{2.5}\right)^2 = 0.04$

Diamond / air $\quad n_1 = 1 \quad n_2 = 2.63 \quad R = 0.2 \ (20\%)$

We can understand a lot about reflection and transmission by analyzing the expressions for r & t known as Fresnel's equations

Two polarizations are reflected differently (very differently!)



If $n_1 < n_2$, then $\theta_1 > \theta_2$ and $\cos\theta_1 < \cos\theta_2$

$\underline{r_\perp}$: Thus $n_1\cos\theta_1 > n_2\cos\theta_2$ for any incident angle (and $r_\perp < 0$, which indicates an additional phase shift by π)

At the same time for $r_{\parallel}$: at a particular angle $n_2\cos\theta_1 = n_1\cos\theta_2$, and $r_{\parallel} = 0$!

By using Snell's law $n_1\sin\theta_1 = n_2\sin\theta_2$, one can show that $\theta_1 = \theta_{Brewster}$

$$\tan\theta_B = \frac{n_2}{n_1}$$

Proof:

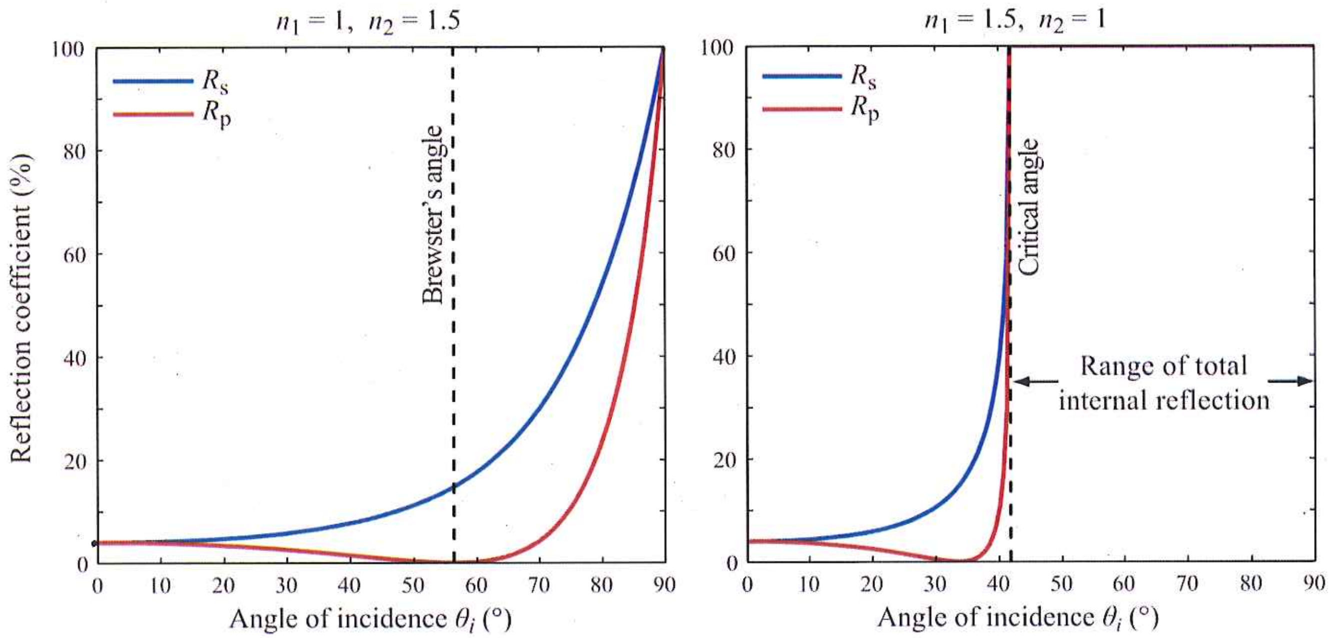
$$n_2^2\cos^2\theta_B = n_1^2\cos^2\theta_2$$

$$n_2^2(1-\sin^2\theta_B) = n_1^2(1-\sin^2\theta_2) = n_1^2\left(1-\frac{n_1^2}{n_2^2}\sin^2\theta_B\right)$$

$$n_2^4(1-\sin^2\theta_B) = n_1^2(n_2^2 - n_1^2\sin^2\theta_B)$$

$$\sin^2\theta_B = \frac{n_2^2}{n_1^2+n_2^2} \qquad \cos^2\theta_B = 1-\sin^2\theta_B = \frac{n_1^2}{n_1^2+n_2^2}$$

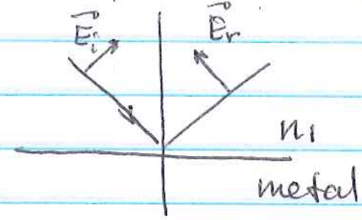
$$\tan\theta_B = \sqrt{\frac{\sin^2\theta_B}{\cos^2\theta_B}} = \frac{n_2}{n_1}$$



$n_1 = 1, \ n_2 = 1.5$
R_s
R_p
Brewster's angle
Reflection coefficient (%)
Angle of incidence θ_i (°)
$n_1 = 1.5, \ n_2 = 1$
Critical angle
Range of total internal reflection
0
10
20
30
40
50
60
70
80
90
100

Why metals reflect light?

Electric field inside a metal must be zero, but Maxwell's equs still work

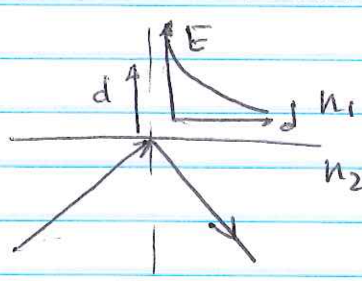


$$\underbrace{E_i \cos\theta_1 - E_r \cos\theta_1}_{\text{air side}} = \underset{\text{metal side}}{0}$$

$$E_i = E_r \ !$$

Light has to reflect!

What about total internal reflection?



Evanescent field (electromagnetic wave that decays very rapidly

Because of the same boundary conditions k becomes imaginary!

$$\vec{k} \rightarrow i k \qquad e^{ikz} \rightarrow e^{-kz} = e^{-2\pi z/\lambda}$$

The wave decays at a distance of $\sim \lambda$

Still, some of light energy is outside, and because of it the reflected light is affected by the properties of the "forbidden" region