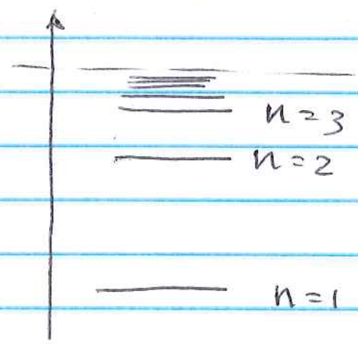
Interaction of light with atoms: semiclassical description

semiclassical = quantized atoms + classical e-m field

From ~~quantum~~ ~~class~~ quantum mechanics: descrete spectrum of electron states
Hydrogen atom (exactly solvable)
everything elese – ~~ap~~ H-atom + corrections



$$E_n = -\frac{R_y}{n^2} \qquad R_y = 13.6\,\mathrm{eV}$$

Let's recall our classical oscillator model for a moment: atomic responce ⇒ ~~refr~~ polarizability

$$\cancel{n} = \cancel{1}\ \alpha(\omega) = \frac{\cancel{\omega_p^2}\, e^2}{\varepsilon_0 m}\,\frac{1}{\omega_0^2-\omega^2+i\gamma\omega}$$

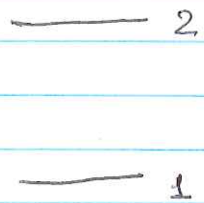
and, for $\gamma \ll \omega_0$

$$\alpha(\omega) \simeq \frac{\cancel{\omega_p^2}}{\cancel{2}}\,\frac{e^2}{2\varepsilon_0 m\omega_0}\,\frac{1}{\omega_0-\omega+i\gamma/2}$$

Strong response only for $\omega-\omega_0 \sim \gamma$
if ω is very different from ω_0 $\alpha \to 0$

That behavior is also true for quantum system; the light interacts with electron stronger if its frequency is close to the energy difference b/w some two transitions. $\omega_0 \to (E_f - E_i)/\hbar$

That also means that even though there are many energy levels in atoms, only the two that are in resonant conditions ($\omega \sim \omega_0$) are going to be important $\Rightarrow$ Two-level approximation



Frequency of an atomic transition
$\omega_0 = (E_2 - E_1)/\hbar$
Frequency of an e-m field
ω, close to ω_0

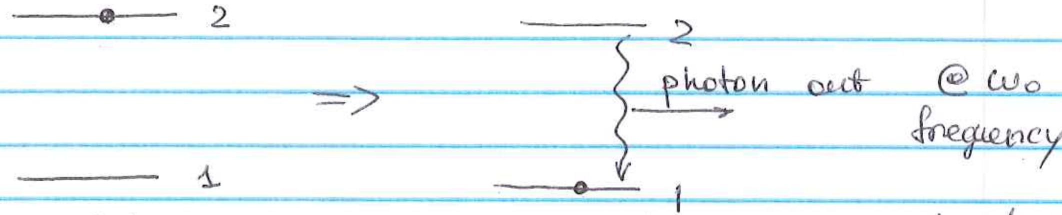
Quantum state of atoms
$$|\psi\rangle = c_1|1\rangle + c_2|2\rangle$$
Probability ~~of~~ to find an atom in state 1 $|c_1|^2$, and in state 2 $- |c_2|^2$
If we have many identical atoms (N), then $N_1 = N \cdot |c_1|^2$ will be found in state 1, and $N \cdot |c_2|^2$ - in state 2.

In a thermal equilibrium $\cancel{\frac{N_2}{N_1}}$ $N_i \sim e^{-E_i/k_BT}$
and $\frac{N_2}{N_1} = e^{-(E_2-E_1)/k_BT}$

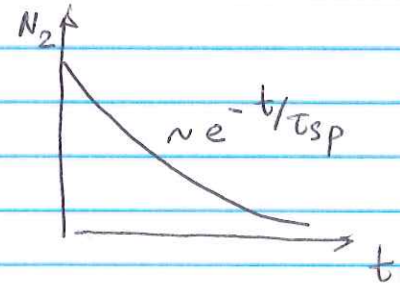
typically most atoms are in lower-energy state. In fact, in optical transitions the higher states are basically empty
$k_BT \approx 0.03\,eV$ and $E_2 - E_1 \sim 1\,eV \gg k_BT$

Three important light-atom interactions

Spontaneous emission



$$\frac{dN_2}{dt} = -A_{21} \cdot N_2 \qquad N_2 = N_2(t=0)\, e^{-A_{21} \cdot t}$$

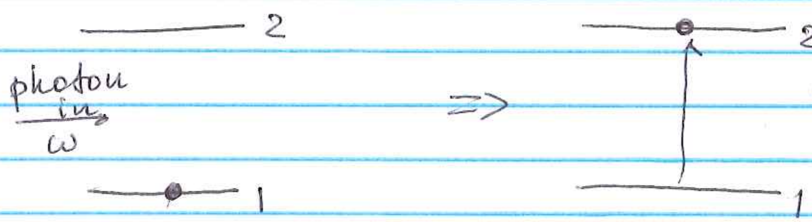


Spontaneous lifetime of the excited state

$$N_2 = N_2(t=0)\, e^{-t/\tau_{sp}}$$

$$\tau_{sp} = 1/A_{21}$$

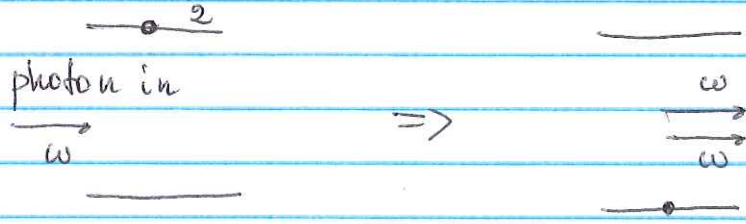
Stimulated absorption



photon is absorbed atom is in the excited state

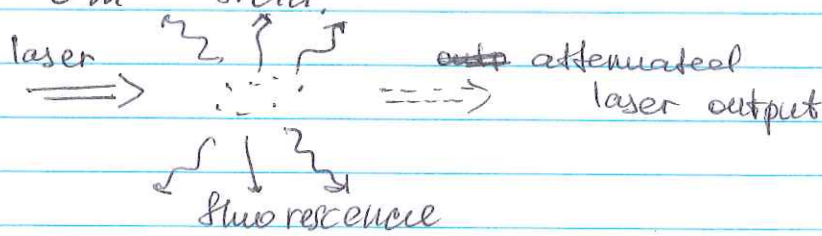
$$\frac{dN_2}{dt} = B_{12} N_1 \cdot \underbrace{W(\omega)}_{\text{spectral energy density of e-m wave}}$$

Stimulated emission



$$\frac{dN_2}{dt} = -B_{21} N_2\, W(\omega)$$

Regular atoms: all atomic population at the ground state, no atoms in the excited state
- absorption is the strongest
(starts populating state 2)
But if the field is weak, so that stimulated emission is not strong enough, the spontaneous emission overtakes, ~~the~~ and energy is dissipated via spontaneous emission from the incoming e-m field.



A_{21}, B_{12} and B_{21} – Einstein coefficient

$$B_{12} = B_{21}, \quad \text{and} \quad \frac{A_{21}}{B_{21}} = \frac{8\pi\hbar f^3}{c^3} \qquad f = \omega/2\pi$$

$1/A_{21} = \tau_{sp}$ → lifetime of an excited quantum state, can be calculated using advance quantum mechanics.

Can we amplify the incoming e-m wave?
Yes! If stimulated emission is the dominent process
Assuming spontaneous emission negligeable for e-m field travelling from z to $z+dz$

dt

photon loss ~~gain~~: $dN^{(-)}_{photons} = -dN_2 = -B_{12} N_1 W(\omega) \, dz/c$

photon gain ~~loss~~: $dN^{(+)}_{photon} = -dN_2 = B_{21} N_2 W(\omega) \, dz/c$

Each photons carries energy $\hbar\omega$

Total energy balance

$$dW = \hbar\omega \left(B_{21} N_2 W - B_{12} N_1 W \right) \cdot dz/c =$$

$$= \frac{\hbar\omega}{c} B_{21} (N_2 - N_1) W \; dz$$

$$\frac{dW}{dz} = \frac{\hbar\omega}{c} B_{21} (N_2 - N_1) W$$

$$W(z) = W(z=0)\, e^{gz} \qquad g - \text{gain}$$

$$g(f) = A_{21} (N_2 - N_1) \frac{c^2}{2\pi f^2} F(f)$$ spectral line of the absorption

To achieve gain need $N_2 > N_1$
population inversion

LASER – light amplification by the Stimulated Emission of Radiation