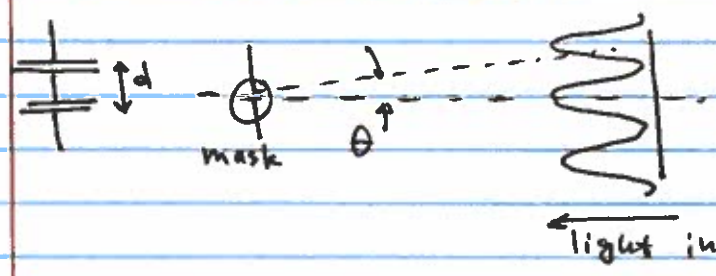
Diffraction & (or) vs Interference

Interference — sum of two or several distinct waves

Two-slit interference with narrow slits ($w \ll \lambda$)

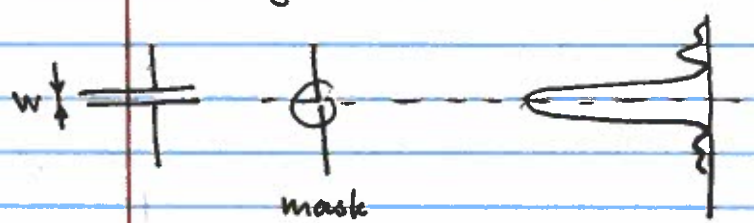


Bright fringe
$\theta_m^{(max)} = m \lambda/d$
Dark fringe
$\theta_m^{(min)} = (m+\frac{1}{2}) \lambda/d$

General intensity pattern

$$I_{2slit}(\theta) = I_0 \cos^2\left(\frac{\pi d \theta}{\lambda}\right)$$

Single-slit diffraction



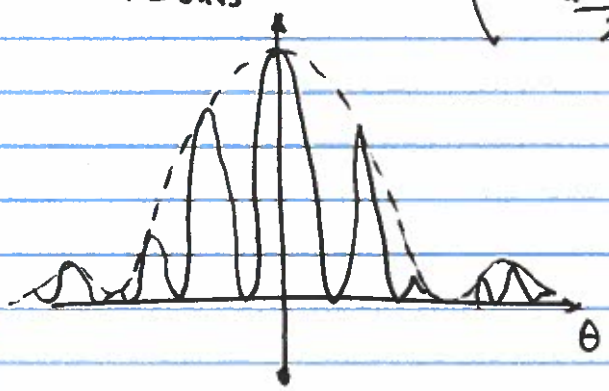
First dark line
$\theta_1^{(min)} = \lambda/w$
m-th minimum
$\theta_m^{(min)} = m\lambda/w$

General intensity pattern

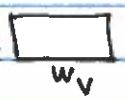
$$I_{single\ slit}(\theta) = I_0 \left(\frac{\sin \frac{\pi w \theta}{\lambda}}{\frac{\pi w \theta}{\lambda}}\right)^2$$

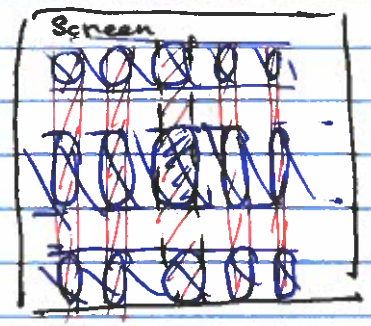
How's interference pattern from the two slits of width w

$$I^{(\theta)}_{\text{real 2 slits}} = I_0 \left(\frac{\sin \frac{\pi w \theta}{\lambda}}{\frac{\pi w \theta}{\lambda}} \right)^2 \cos^2 \left(\frac{\pi d \theta}{\lambda} \right)$$

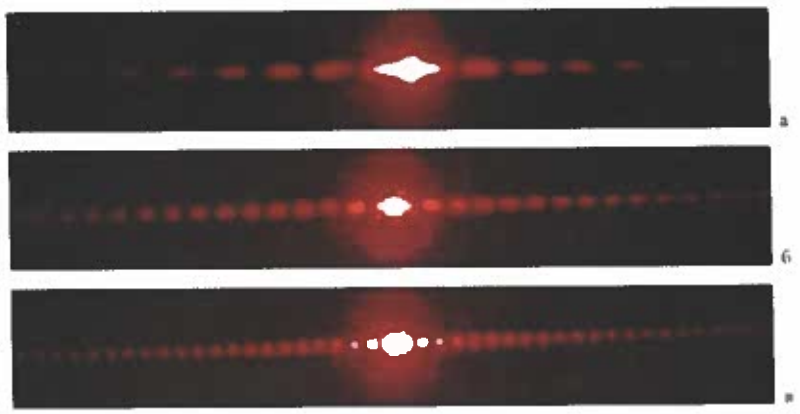


Two - D slits

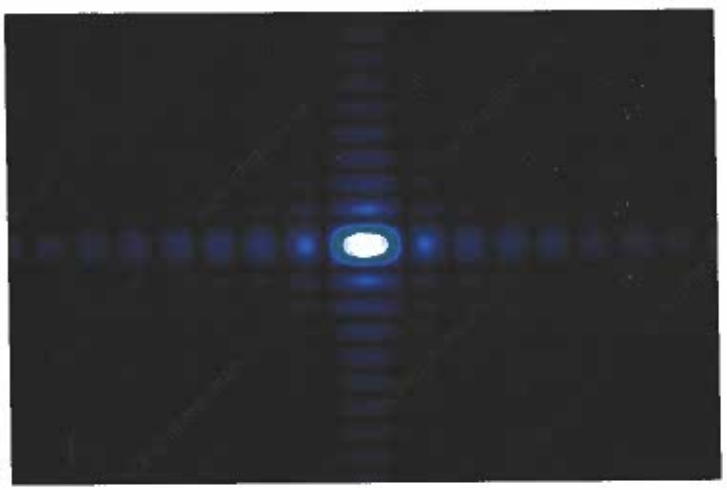
Aperture 

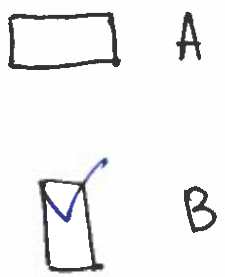


The shape of the rectangle is inverted, since the divergence is inversly proportional to the slit width.



Which slit is wider? (top, middle, bottom)

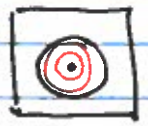




What is the shape of the slit?

Is the horizontal size smaller or larger than the vertical size?

Round aperture



We can calculate the total field amplitude ~~one~~ at each θ. the result is calculatable, but involve special Bessel function. Important take away — the image of the circular aperture of diameter D has the central spot divergence

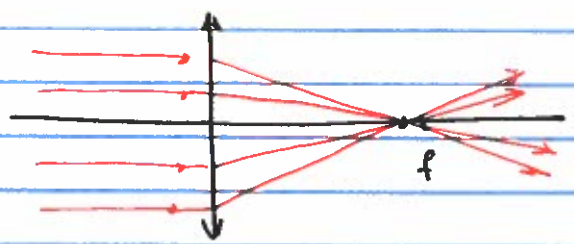
$$\theta = 1.22 \frac{\lambda}{D}$$

So the size of the spot at the screen at distance L is

$$\langle \text{spot diameter} \rangle = 1.22 \frac{\lambda \cdot L}{D}$$

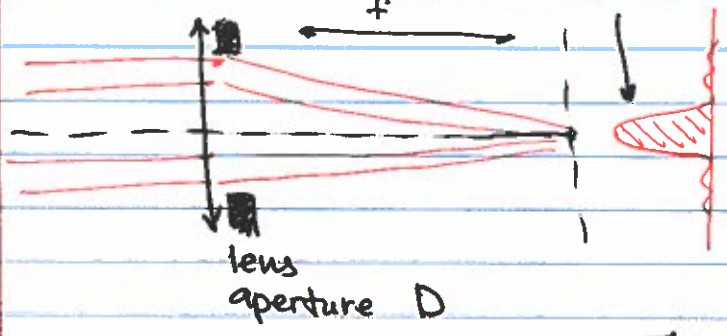
Given that most of optics is circular, this expression appears in many situations, this expression is often use to estimate the achievable resolution of optical system

Ray optics



In principle, with an "ideal" lens we should be able to focus all rays in the perfect poin

More realistic view



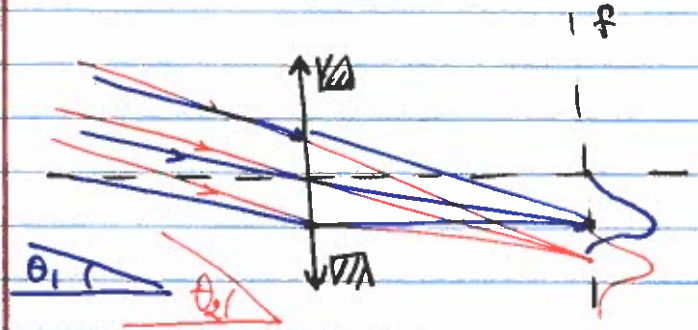
Minimum radius of the focal spot

$$r_{min} = \frac{1.22 \lambda f}{D}$$

Diffraction limit

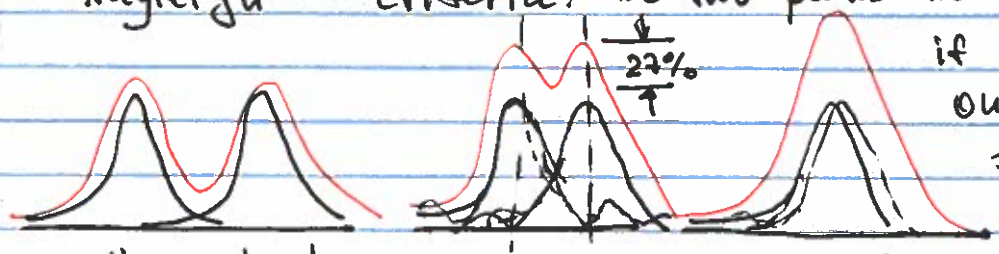
The min spot is smaller for smaller f, but it is larger for smaller D.

Angular resolution (our old friend from telescope discussion)



The diffraction-limited spot size will determine how small an angle one can resolve

Rayleigh criteria: the two peaks are resolved if the top of one is on top of the first diffraction minimum for the other



well-resolved unresolved

$\theta_{min} = 1.22 \lambda / D$ minimum angle resolution

Question: what is an angular resolution of a telescope with lens diameter $D = 10\,cm$?

$$\theta_{min} = 1.22\,\frac{\lambda}{D} = 1.22\,\frac{500nm}{10cm} = 1.22\,\frac{5\cdot10^{-7}\,m}{10^{-1}\,m} =$$

$$= 6\cdot10^{-6}\ rad$$

Can we use this telescope to resolve the two α-Centauri stars?

Average separation 23 AU

Distance to Earth 270,000 AU

Angular separation $\theta_{dcent} = \frac{23}{270000} = 8.5\cdot10^{-5}\ rad$

Since $\theta_{dcent} > \theta_{min}$ of the telescope, it is resolvable

What about human eye?

$$\theta_{min}^{(eye)} = 1.22\,\frac{500nm}{5mm} = 1.22\,\frac{5\cdot10^{-7}m}{5\cdot10^{-3}m} = 1.2\cdot10^{-4}\ rad$$

not enough to resolve α-Centauri double star