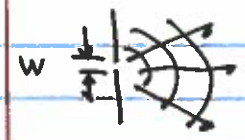
Diffraction

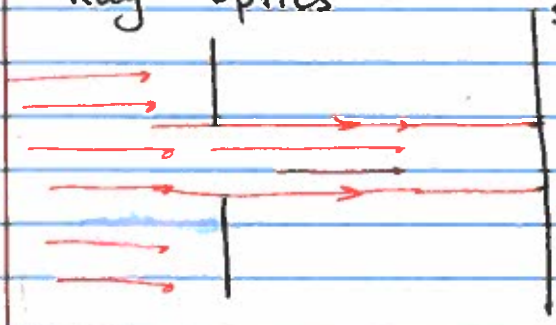
Technically, diffraction is a type of interference phenomena, in which light from different parts of an opening interfere

If a slit is very small $w \ll \lambda$, we can treat it as a point source



For a wider slit, we need to account for phase difference b/w different parts (especially near the edge)

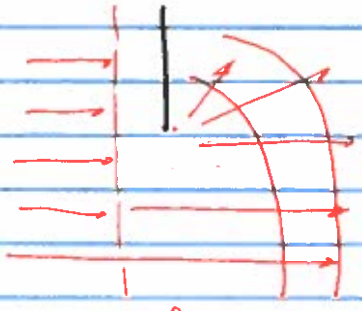
Ray optics





expect sharp edge / shadow on the screen

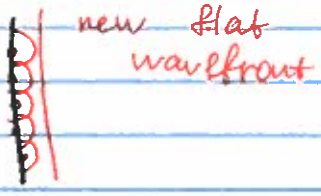
Let's look more carefully what happens at the edge



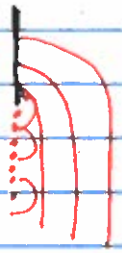
Because of the edge, the wavefront is bending, so not all light travels straight

To better understand diffraction, we need to apply Huygens - Fresnel principle: (from 1678)
every point on a wavefront serves as a source of secondary wavelets, and the new wavefront is the common envelope of all these wavelets

Plane wave



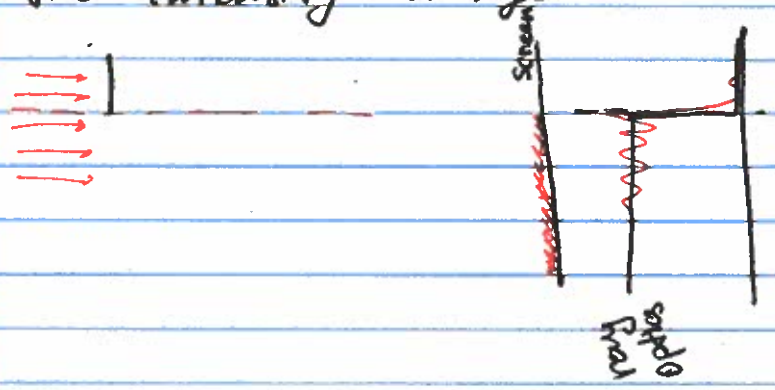
Edge



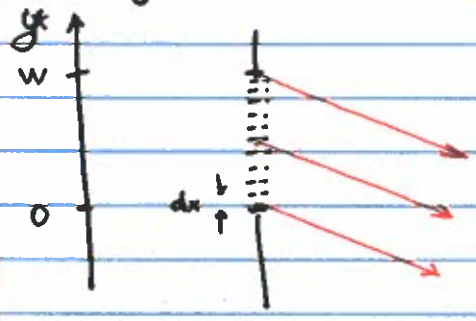
The wavelets at the edge cause curving of the wavefront, making it "spill" beyond the boundary

The optical field in every point beyond the obstruction is the superposition of all wavelets reaching that points

If the opening is large, most of it is not affected by the interference at the edges, but near the edge diffraction creates an "interferance patterm", smoothing the intensity change



Single-slit diffraction (one-dimentional case)



$$dE(\theta) = E_0 \frac{dy}{w} \cos(kx - \omega t + ky\cdot\theta)$$

electric field of one wavelet

$$E_{tot} = \Sigma dE(\theta) = \int_0^w dE(y,\theta)$$

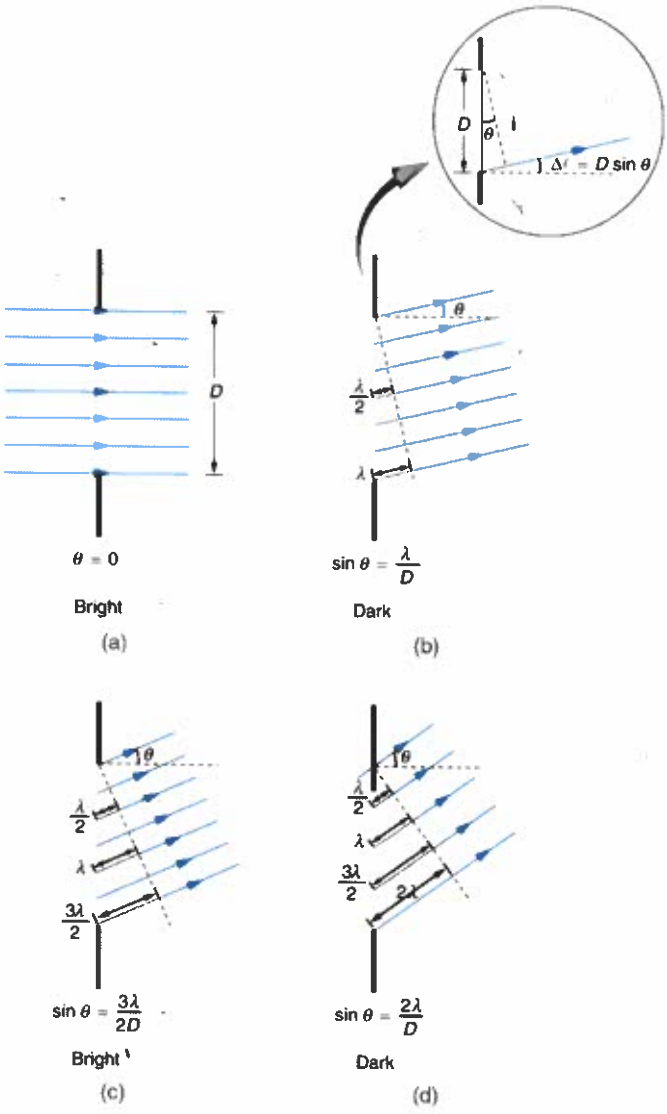
$$= \int_0^w \frac{E_0}{w} dy \cdot \cos(kx - \omega t + ky\cdot\theta) = \frac{E_0}{w} \int_0^w \cos(kx - \omega t + ky\, dy)$$

$$= \frac{E_0}{w} \frac{1}{k\theta} \left[\sin(kx - \omega t) - \sin(kx - \omega t + kw\theta) \right]$$

$$= \left[\frac{2E_0}{kw\theta} \sin\frac{kw\theta}{2} \right] \cos\left(kx - \omega t + \frac{kw\theta}{2}\right)$$

amplitude

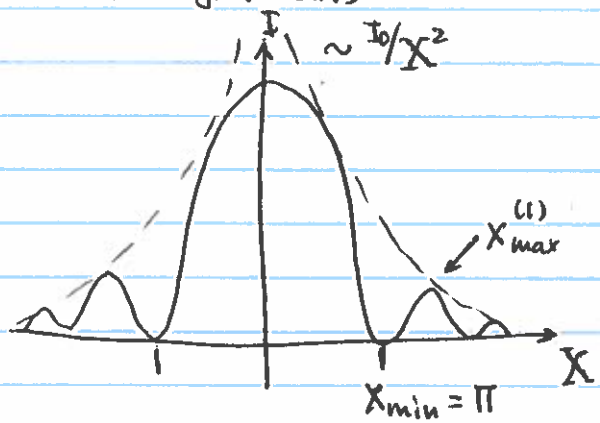
Intensity $$I = \frac{\varepsilon_0 c}{2} 4E_0^2 \left(\frac{\sin\frac{kw\theta}{2}}{kw\theta} \right)^2$$


D
θ
Δl = D sin θ
θ = 0
Bright
(a)
θ
λ/2
λ
sin θ = λ/D
Dark
(b)
θ
λ/2
λ
3λ/2
sin θ = 3λ/2D
Bright
(c)
θ
λ/2
λ
3λ/2
2λ
sin θ = 2λ/D
Dark
(d)

More general expression for the single-slit diffraction

$$I = I_0 \left(\frac{\sin X}{X}\right)^2 \quad \text{where} \quad X = \frac{kw\sin\theta}{2} \approx \frac{kw\theta}{2}$$

(I_0: total intensity through the slit; for small angles)

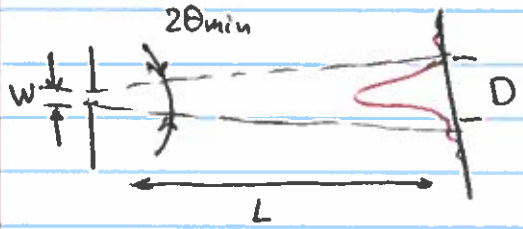


$$X_{min} = \frac{kw\theta_{min}}{2} = \pi$$

$$\frac{1}{2}\cdot\frac{2\pi}{\lambda}\cdot w\,\theta_{min} = \pi$$

$$\theta_{min} = \frac{\lambda}{w}$$

The smaller is the slit width, the larger is divergence



The size of the spot on the screen

$$D = w + 2L\cdot\theta_{min} = w + \frac{2L\lambda}{w}$$

If $w = 0.1\,mm = 10^{-4}\,m$ $\quad \theta_{min} = \frac{5\cdot10^{-7}m}{10^{-4}m} \approx 5\cdot10^{-3}\,rad \approx 0.3°$

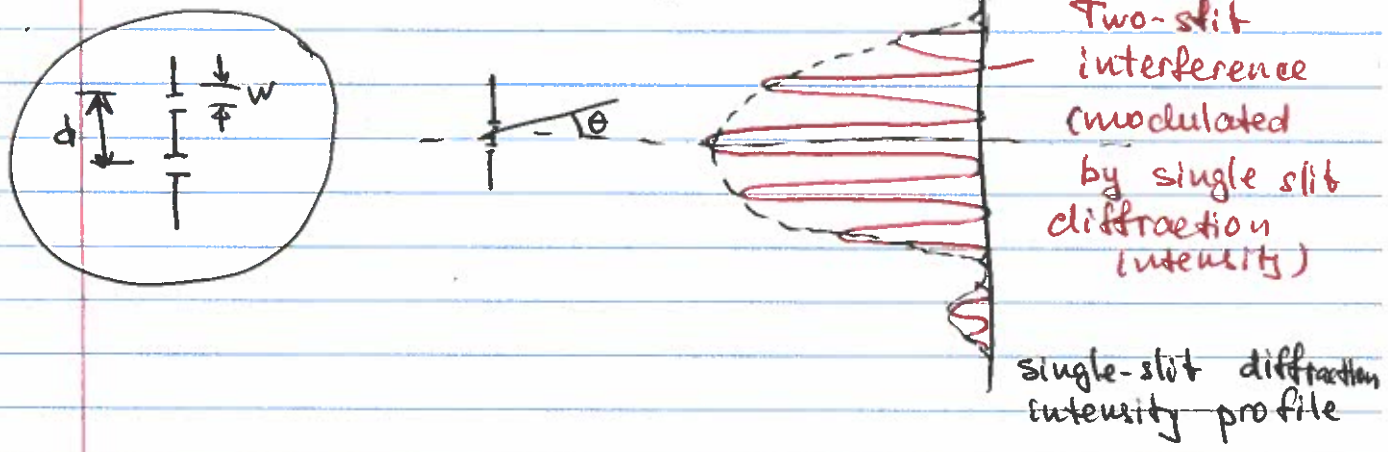
$D = 10^{-4}m + 5\cdot10^{-3}\cdot 2m = 10^{-4}m + 10^{-2}m \approx 10^{-2}m = 1cm$

Next maximum $\quad X = 3\pi/2$

$$\frac{\pi}{\lambda}\cdot w\,\theta^{(1)}_{max} = 3\pi/2 \qquad \theta^{(1)}_{max} = \frac{3\lambda}{2w}$$

The distance b/w the center and the first max $\quad d_{max} = L\cdot\theta^{(1)}_{max} = 3/2\ L\lambda/w$

More realistic description of the two slit interference profile



Mathematically

$$I(\theta) = I_0 \left(\frac{\sin \frac{\pi w \theta}{\lambda}}{\frac{\pi w \theta}{\lambda}} \right)^2 \cdot \cos^2 \frac{\pi d \theta}{\lambda}$$

single-slit diffraction — two-slit interference

Similarly, for a diffraction grating

$$I(\theta) = I_0 \left(\frac{\sin \frac{\pi w \theta}{\lambda}}{\frac{\pi w \theta}{\lambda}} \right)^2 \left(\frac{\sin \left(\frac{N \pi d \theta}{\lambda} \right)}{\sin \left(\frac{\pi d \theta}{\lambda} \right)} \right)^2$$

single-slit diffraction — ~~multi~~ N-slit interference