30 sec version of electromagnetism

Maxwell's equations describe all relationship b/w electric field, magnetic field, electric charges and electric currents

in vacuum

$$\begin{cases} \nabla \cdot \vec{E} = \rho/\varepsilon_0 \\ \nabla \cdot \vec{B} = 0 \\ \nabla \times \vec{E} = -\partial \vec{B}/\partial t \\ \nabla \times \vec{B} = \mu_0 \vec{J} + \mu_0 \varepsilon_0 \frac{\partial \vec{E}}{\partial t} \end{cases}$$

$\nabla = \hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}$ gradient vector

Translation:

(1) Electric charges create electric field

(2) There is no magnetic charges

(3) If magnetic field changes in time, it produces electric field (perpendicular)

(4) Magnetic field is created either by electric current or by time-varying electric field (perpendicular)

Story of light creation and interaction with electric charges

Light ⇒ electromagnetic field ⇒ time/space varying electric and magnetic fields forming a wave



a charge (oscillating) → generates e-m wave

↑ electric force
• yanking the charge
↓ back and ~~force~~ (becomes a secondary source of EM wave)

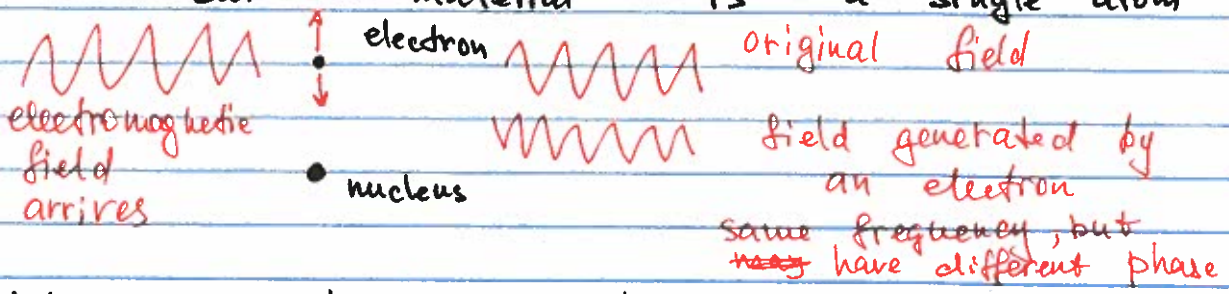
Interaction of ~~elect~~ optical electromagnetic wave with materials

In these calculations we assume that original optical field is originated somewhere else, and arrives to the material

What electrical charges we have inside a material ?

Electrons, ~~nucleu~~ (too heavy)
strongest interaction — with electric fields

If our "material" is a single atom



What can happen with electromagnetic wave in this interaction?

1. It can loose energy, if some work is done moving electrons around
   → absorption, our field energy → brightness →
   → amplitude becomes smaller

2. The wave phase / speed can change, since now we need to consider the interference b/w the original wave and the secondary wave emitted by the atom

In metals electrons can move freely, grab the energy from electromagnetic field and "waste it" on kinetic energy. That is why metals are very absorbing, no light travels through metal (but they make good mirrors!)

Dialectrics can be transparent — all electrons are "locked" to atoms and cannot move much, only wiggle in responce to passing electromagnetic wave. That creates an additional field due to re-emission

Total electric field (usually called $\frac{1}{\varepsilon_0}\vec{D}$)

$$\frac{1}{\varepsilon_0}\vec{D} = \vec{E} + \frac{1}{\varepsilon_0}\vec{P}$$

$\vec{E}$: original electric field

$\frac{1}{\varepsilon_0}\vec{P}$: polarization of the medium, reradiated electric field

$\vec{P}$ will contain information about atomic / molecular / material response to the presence of electromagnetic field

For a medium consisting of identical atoms / molecules,

$$\vec{P} = N \cdot \vec{d}$$

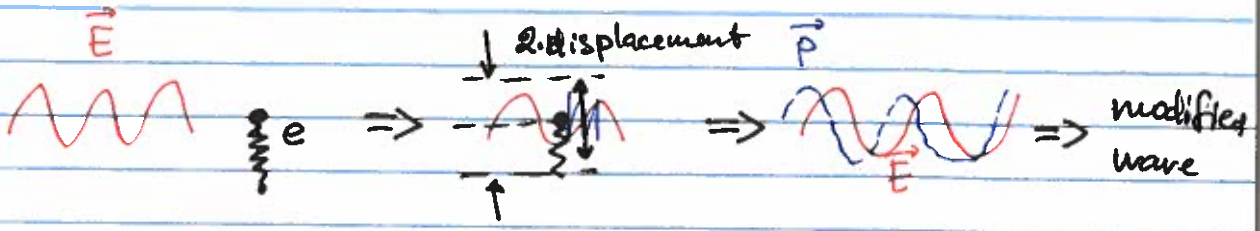
$\vec{d}$ = dipole moment

$$\vec{d} = e \cdot \vec{r}$$

e: charge of an electron

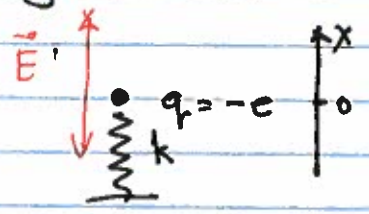
$\vec{r}$: displacement from the equilibrium

Cartoon of light atom interaction



How to calculate the electron oscillations?

Toy model



Electric field provide an oscillating force

$E = E_0 \cos \omega t$

$F_{el} = (-e) E_0 \cos \omega t$

$F_{atom} \approx -kx$
like a spring

Newton's law

$m\vec{a} = F_{electric} + F_{atom}$

$m \frac{dx^2}{dt^2} = -eE_0 \cos\omega t - kx$

$x = x_0 \cos \omega t \qquad \frac{dx^2}{dt^2} = -x_0 \omega^2 \cos \omega t$

$-m x_0 \omega^2 \cos\omega t = -eE_0 \cos\omega t - k \cdot x_0 \cos\omega t$

$m x_0 \omega^2 - k x_0 = eE_0$

$x_0 = \frac{eE_0}{m\omega^2 - k} = \frac{eE_0}{m} \frac{1}{\omega^2 - \omega_0^2}$

$\omega_0 = \sqrt{k/m}$

"natural" Resonant frequency

$$x = \underbrace{\frac{eE_0}{m}\frac{1}{\omega^2 - \omega_0^2}}_{x_0}\cos\omega t$$

$$P(t) = \underbrace{N\cdot(-e)\frac{eE_0}{m}}_{\text{depends on atoms}}\boxed{\frac{1}{\omega^2-\omega_0^2}}\underbrace{E_0\cos\omega t}_{\text{electric field}}$$

resonance responce

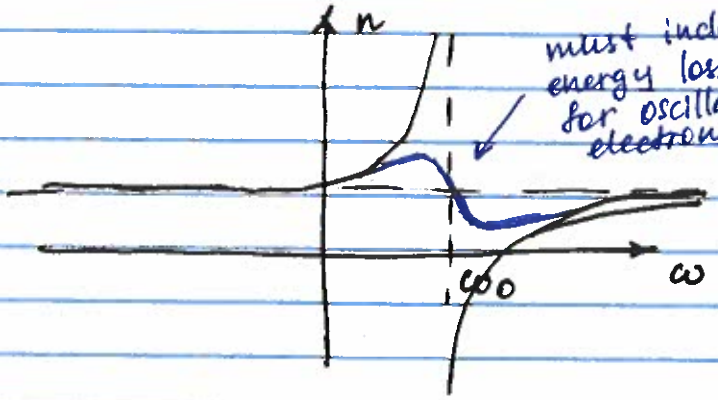
Polarization is proportional to electric field

$\vec{P} = \alpha\vec{E}$ α - polarizability

$$n \approx 1 + \frac{1}{2}N\frac{\alpha}{\varepsilon_0} \approx 1 + \frac{1}{2}\frac{e^2}{\varepsilon_0 m}\cdot N\frac{1}{\omega_0^2-\omega^2}$$

$$= 1 + \frac{1}{2}\omega_p\frac{1}{\omega_0^2-\omega^2}$$

$$\boxed{\omega_p = N\frac{e^2}{\varepsilon_0 m}}$$ plasma frequency



$n \approx 1$ for most of ω but changes a lot around ω_0

More realistic in blue