Problem set #8 (solutions)

A1: The intensities of two e-m waves add up when the waves are perpendicularly polarized.

$$\vec{E}_1 = \vec{E}_{1x} + \vec{E}_{1y}$$
$$\vec{E}_2 = \vec{E}_{2x} + \vec{E}_{2y}$$

$$I_{total} \propto \langle (\vec{E}_1 + \vec{E}_2)^2 \rangle = \langle (E_{1x} + E_{2x})^2 + (E_{1y} + E_{2y})^2 \rangle$$

if $E_{2x} = 0$ and $E_{1y} = 0$

$$I_{total} = \langle E_{1x} \rangle^2 + \langle E_{2y} \rangle^2 = I_1 + I_2$$

The relative phase b/w the two waves does not matter in this case.

A2

Elliptically polarized light

$$\vec{E}(z,t) = E_1 \vec{e}_x \cos(kz - \omega t) + E_2 \vec{e}_y \sin(kz - \omega t) =$$

$\{$ let's assume for convenience that $E_1 < E_2$ $\}$

$$= E_2 \vec{e}_x \cos(kz - \omega t) + E_2 \vec{e}_y \sin(kz - \omega t) + (E_1 - E_2)\vec{e}_x \cos(kz - \omega t) =$$

$$= \underbrace{E_2 (\vec{e}_x \cos(kz - \omega t) + \vec{e}_y \sin(kz - \omega t))}_{\text{circularly polarized wave with amplitude } E_2} + \underbrace{(E_1 - E_2)\vec{e}_x \cos(kz - \omega t)}_{\text{linearly polarized wave with amplitude } (E_1 - E_2)}$$

A3.

Malus law: $I_{out} = I_{in} \cos^2 \varphi$

For ~~A~~ a polarizer at $45°$ the transmission is $\cos^2 45° = 1/2$

That means that $1/2\, I_1 = \frac{1}{4} I_0$ passes through the first and the third polarizers, and then $1/2$ of that $= 1/8\, I_0$ passes through the second polarizer

If the middle (third) polarizer is rotating, its angle with the axis of the first polarizer is ωt, so its transmission is $\cos^2 \omega t$. Thus, the intensity after the first and the third polarizers is

$$I_3 = \frac{1}{2} I_0 \cos^2 \omega t = \frac{1}{4} I_0 (1 + \cos 2\omega t)$$

Because the second polarizer is $90°$ off compare to the first one, the angle b/w its axis and the rotating perarizer is $\omega t + 90°$, so its transmission is $\cos^2(\omega t + 90°) = \sin^2 \omega t$

Thus, the total intensity after the second polarizer is $I_2 = I_3 \cdot \sin^2 \omega t =$

$$= \frac{1}{2} I_0 \cos^2 \omega t \sin^2 \omega t = \frac{1}{8} I_0 \sin^2 2\omega t = \frac{1}{16} I_0 (1 - \cos 4\omega t)$$

Trig. identities: $2 \sin x \cos x = \sin 2x$

$2 \sin^2 x = 1 - \cos 2x$

10.5

The intensity after the first polarizer is $\frac{1}{2}I_0$, and rotating, following the rotation axis of the polarizer itself.

The relative angle b/w the two polarizer axes is $2\omega t$, so the transmission of the second polarizer is $\cos^2(2\omega t)$, and the transmitted intensity is

$$\underline{I_2 = \frac{1}{2}I_0 \cos^2 2\omega t} = \frac{1}{4}I_0 (1 + \cos 4\omega t)$$

The transmitted polarization direction is determined by the axis of the second polarizer, so it will rotate with frequency ω (even though the intensity oscillates at the frequency of (4ω).