Homework #5 (solutions)

A1 $\lambda = \frac{c}{f} = \frac{3 \cdot 10^{8} m/s}{7 \cdot 10^{15} 1/s} = 4.3 \cdot 10^{-8} m = 43\ nm$

Surface irregularities $\lambda/8 \approx 5.5 nm$
This is UV light [illegible]
(due to a typo in the frequency value), so
it is invisible

A2

An individual source $E_1 = E_0 \cos(kz - \omega t)$

$I_1 = \langle E_1^2 \rangle = \frac{1}{2} E_0^2$

Two interfering sources $E = \left(2E_0 \cos\frac{\varphi}{2}\right) \cos(kz - \omega t + \varphi/2)$

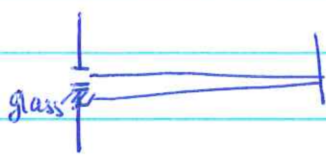
$I_2 = 2E_0^2 \cos^2 \varphi/2$

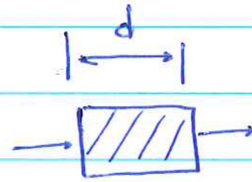
For $I_1 = I_2$

$2E_0^2 \cos^2 \varphi/2 = \frac{1}{2} E_0^2 \Rightarrow \cos^2 \varphi/2 = 1/4$

$\cos \varphi/2 = 1/2, \quad \varphi = \frac{2\pi}{3} (120°)$

A3





phase of light after travelling
- through the glass $\frac{2\pi n}{\lambda} \cdot d$
through the air $\frac{2\pi}{\lambda} d$

For destructive interference

$$\frac{2\pi n}{\lambda} d - \frac{2\pi}{\lambda} d = \frac{2\pi(n-1)d}{\lambda} = \pi \text{ or } 3\pi \text{ or } 5\pi$$

$d_{min} = \frac{\lambda}{2(n-1)} = \lambda = 500 nm$

$d_2 = \frac{3\lambda}{2(n-1)} = 3\lambda = 1500 nm$

$d_3 = 5\lambda/2(n-1) = 5\lambda = 2500 nm$

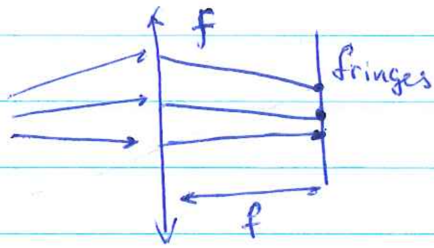
A4

$d = 0.02 mm$

$\lambda = 633 nm$

Angular difference b/w consecutive maxima

$d \cdot \theta = \lambda$ (assuming small angle $\theta \approx \sin\theta$)



$$\Delta x = f \cdot \theta = f \frac{\lambda}{d} = 1.5m \cdot \frac{633 \cdot 10^{-9} m}{2 \cdot 10^{-5} m}$$

$$\Delta x = 0.047m = 47 mm$$

Bonus

~~For~~ Two light waves of different ~~rate~~ amplitude will display interference, but the destructive interference is not complete

$$E = A \cos(kz - \omega t) + B \cos(kz - \omega t + \varphi) =$$

~~$\sqrt{A^2+B^2}$~~

$$E^2 = A^2 \cos^2(kz - \omega t) + B^2 \cos^2(kz - \omega t + \varphi) +$$
$$+ 2AB \cos(kz - \omega t)\cos(kz - \omega t + \varphi) =$$
$$= A^2 \cos^2(kz - \omega t) + B^2 \cos^2(kz - \omega t + \varphi) +$$
$$+ AB \cos\varphi + AB \cos(2kz - 2\omega t + \varphi)$$

$$I = \langle E^2 \rangle = \frac{1}{2} A^2 + \frac{1}{2} B^2 + AB \cos\varphi \quad (\text{since } \langle \cos(2kz - \omega t) \rangle = 0)$$

$$I_{min} = \frac{1}{2} A^2 + \frac{1}{2} B^2 - AB = \frac{1}{2}(A - B)^2$$

$$I_{max} = \frac{1}{2} A^2 + \frac{1}{2} B^2 + AB = \frac{1}{2}(A + B)^2$$