Homework #10 solutions

11.4

Likely reason for greenish color of the red ink film.

Why the ink is red? It is a solution with particle that absorb all colors except red, so a thick layer of ink does not allow any other colors to propagate.

However, when the layer is thin, it does not produce enough optical depth for full absorption; also, the process of drying may change the size of the particles, ~~to~~ so that the scattered light also become visible. In this case because of the ~~Rayleley~~ Rayleigh scattering the blue/green contribution is stronger, that gives the greenish tint to the film.

Thin film interference is also possible, but a uniform coloring would imply that the thickness of the film is very contant, that is usually hard to achieve.

11.6

Since air has negligeable absorption, the main attenuation is due to Rayleigh scattering

Attenuation coefficient for intensity is

$$\beta = \frac{32}{3}\pi^3 (n-1)^2 \frac{1}{N\lambda^4} = 7.75\cdot 10^{-6}\,m$$

~~Ilass~~ = Attenuation $= e^{-\beta L} = e^{-7.75\cdot 10^{-6} m \cdot 2\cdot 10^{5} m} = 0{,}212$

21% reduction

A1

Absorption coefficient for amplitude

$$\beta = \frac{2\pi}{\lambda} n'' = \frac{2\pi}{632\cdot 10^{-9} m} \cdot 5\cdot 10^{-5} \simeq 497 \ 1/m$$

A2

Four resonances:

$\lambda_1 \simeq 5377 \text{ Å}$ $\quad f_1 = 5.576\cdot 10^{14} Hz$ $\quad \omega_1 = 2\pi \cdot f_1 = 3.504\cdot 10^{15}\ 1/s$

$\lambda_2 \simeq 5387 \text{ Å}$ $\quad f_2 = 5.565\cdot 10^{14} Hz$ $\quad \omega_2 = 3.497\cdot 10^{15}\ 1/s$

$\lambda_3 \simeq 5395 \text{ Å}$ $\quad f_3 = 5.557\cdot 10^{14} Hz$ $\quad \omega_3 = 3.492\cdot 10^{15}\ 1/s$

$\lambda_4 \simeq 5404 \text{ Å}$ $\quad f_4 = 5.548\cdot 10^{14} Hz$ $\quad \omega_4 = 3.486\cdot 10^{15}\ 1/s$

Width of all resonances is similar, approximately 5 Å

$$\lambda = \frac{2\pi c}{\omega} \qquad \Delta\lambda = -\frac{2\pi c}{\omega^2}\Delta\omega \quad (\text{assuming } \Delta\lambda \ll \lambda)$$

$$\Delta\lambda = -\frac{\lambda}{\omega}\Delta\omega \Rightarrow \Delta\omega = -\frac{\Delta\lambda}{\lambda}\cdot\omega$$

Thus for $\frac{\Delta\lambda}{\lambda} \approx 10^{-3}$ $\quad \frac{\Delta\omega}{\omega} \approx 10^{-3}$ ($\Delta\omega = \gamma$)

Resonance full width $\gamma \simeq 10^{-3}\cdot 3.5\cdot 10^{15}\ 1/s$

$\gamma \simeq 3.5\cdot 10^{12}\ 1/s$

A3

$$n(\omega) = 1 + \frac{\omega_p^2}{4\omega}\frac{1}{\omega_0-\omega+i\gamma/2}$$

$$n''(\omega) = \frac{\omega_p^2}{4\omega} Im\left(\frac{1}{\omega_0-\omega+i\gamma/2}\right) = \frac{\omega_p^2}{8\omega}\frac{\gamma}{(\omega-\omega_0)^2+\gamma^2/4}$$

for $\omega \approx \omega_0$ (our region of interest) it is more convenient to write

$$n''(\omega) = \frac{\omega_p^2}{8\omega_0}\frac{\gamma}{(\omega-\omega_0)^2+\gamma^2/4}$$

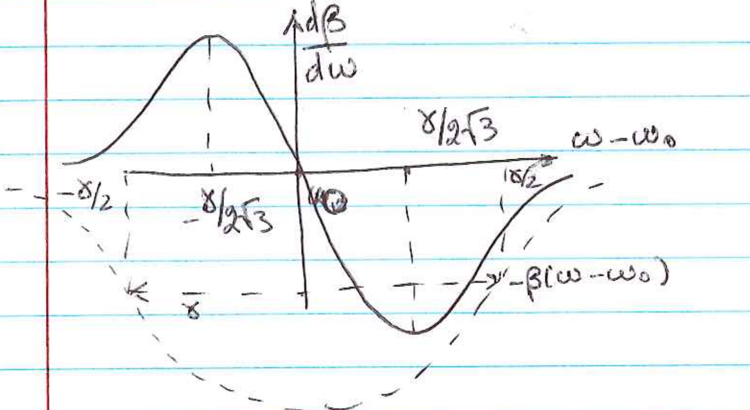
For intensity

$$\beta = \frac{4\pi}{\lambda} n'' = \frac{\pi \omega_p^2}{2\lambda\omega_0} \frac{\gamma}{(\omega-\omega_0)^2+\gamma^2/4} =$$

($\lambda\omega_0 = 2\pi c$)

$$= \frac{\omega_p^2}{4c} \frac{\gamma}{(\omega-\omega_0)^2+\gamma^2/4}$$

$$\frac{\partial\beta}{\partial\omega} = -\frac{\omega_p^2}{4c} \frac{2\gamma(\omega-\omega_0)}{\left[(\omega-\omega_0)^2+\gamma^2/4\right]^2}$$



Peak-to-peak distance $= \frac{\gamma}{\sqrt{3}}$

Bonus: Sugar is optically active, and rotates the polarization of light. However, different colors experience different amount of rotation. (rotation angle $\propto$ frequency) Rayleigh scattering is strongest for perpendicularly polarized light, so the color with the "right" polarization is scattered the most, resulting in rainbow stripes along the tube. They move if the polarizer is rotated!