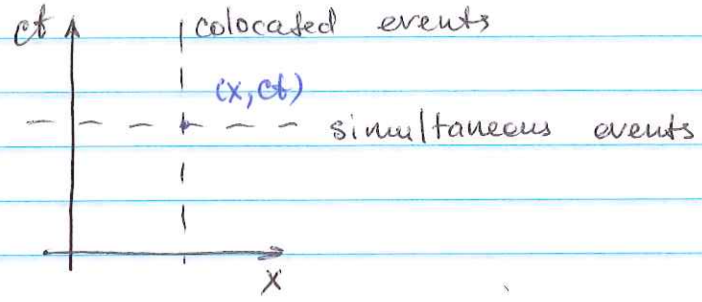
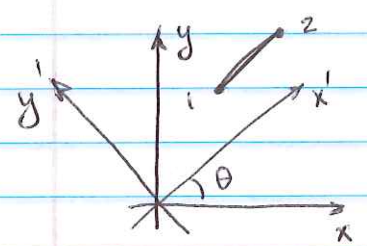
Minkowski space-time (cont)

Each event is characterized by
3 spatial + 1 time coordinate
(x, y, z, ct) (in drawings we usually use only
1 spatial coordinate



Regular space



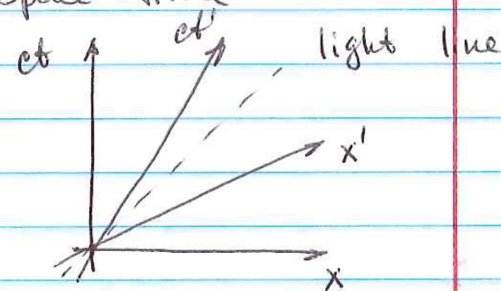
$x' = \cos\theta\, x + \sin\theta \cdot y$

$y' = -\sin\theta \cdot x + \cos\theta \cdot y$

Distance b/w 1 and 2 is invariant

$\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2} = \sqrt{(x_1' - x_2')^2 + (y_1' - y_2')^2}$

Space-time



$x' = \gamma(x - V/c \cdot ct)$ $\quad \gamma = \frac{1}{\sqrt{1 - V^2/c^2}}$

$ct' = \gamma(ct - V/c \cdot x)$

& "Regular" distance $\sqrt{\Delta x^2 + (c\Delta t)^2}$ is no longer invariant.

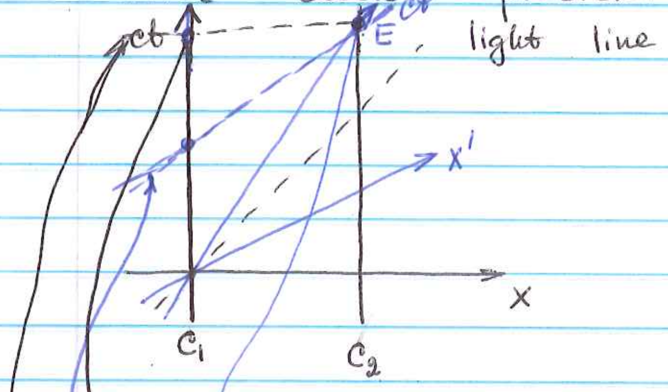
Relativistic invariant

$\sqrt{(x_1 - x_2)^2 - (ct_1 - ct_2)^2} = \Delta S$

(can be real, imaginary or complex

To restore the familiar rules, we need to use four-vectors $(\underline{ict}, x, y, z)$
(but we are not going to use them in this course)

Three-clock problem



E – point in space-time when the shuttle is at the location of C_2

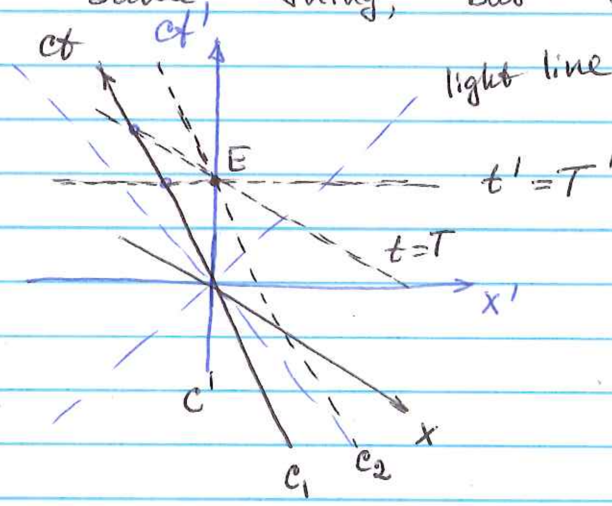
time it happens according to Alice T

time it happens accordind to Bob T'

worldline of all events simultaneous with $t' = T'$

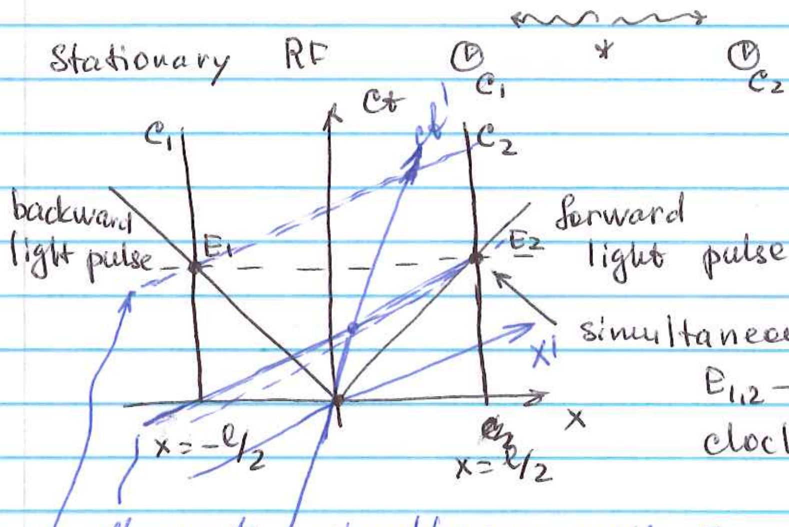
$t = T$

Same thing, but from Bob's perspective



Just like in the case of regular rotation of the coordinate system, any event correspond to a unique point of space-time, but its exact coordinates will depend on RF

Clock synchronization



clock locations $\pm l/2$
pulse emitted at $x=0$, $t=0$

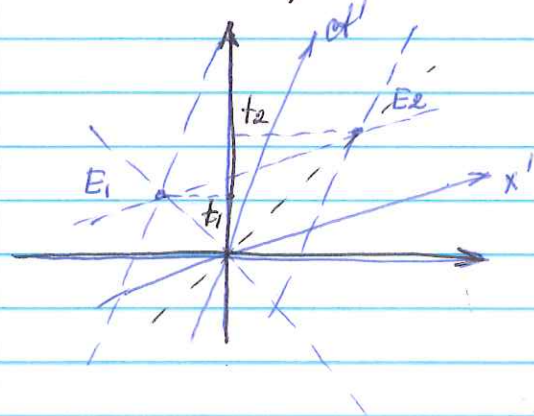
simultaneous events

$E_{1,2}$ – turning on of the clocks C_1, C_2

all events simultaneous with E_2 in the moving RF

all events simultaneous with E_1 in the moving RF (occurs later in this RF)

Alternatively, if the clocks are in the moving RF



$t_1 < t_2$
back clocks start earlier

Right way to approach relativistic kinematics

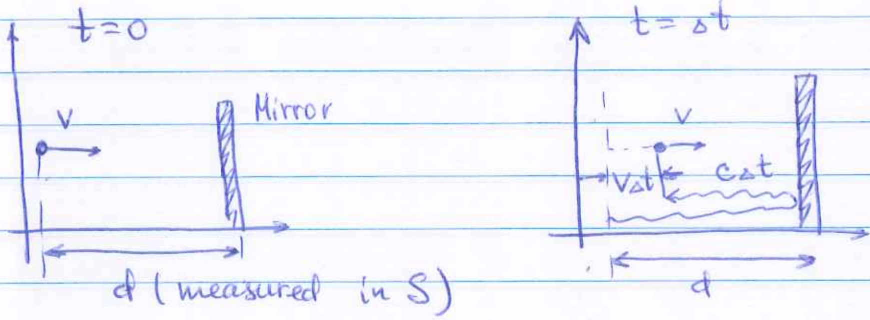
1. Very carefully analize the problem and figure out what parameters and in what RF need to be measured.

2. As long as the calculations are done in the same RF (i.e no measurements made in another RF are ~~needed~~ used), the familiar rules work: all clocks show the same time, regular ~~velocity addition, etc.~~ calculations for durations and length (no γ)

3. When a measurement done in one RF needed to be plugged into the calculations done in anothe RF, relativity becomes necessary, typically in form of a length contraction or time dilation.

4. If it is easy to specify specific events, Lorentz transformations are often the easiest took. However, they can be used only for individual events, to transform its time-space coordinates.
Measurements of length or time interval require two events (start and finish)

Discussion on problem 1.30



To find out how much time has passed in S frame (the frame where the mirror is stationary) we need no relativity! (apart from keeping the speed of light constant)

The time dilation will have to be envoked to find out how much time it passed in ~~Rs~~ the rocket's RF, since now we are changing from one RF to another.

Can we use Lorentz transformation? Sure. Then we need to figure out the space-time coordinates of the ~~event~~ point where the rocket meets the light pulse again. Let's assume that the pulse was originally emitted at $t=0$, $x=0$.
Then when it hits the rocket again

$$t = \Delta t, \quad x = v\Delta t$$

And we can transform them to t' to find out how much time has passed in the rocket's RF