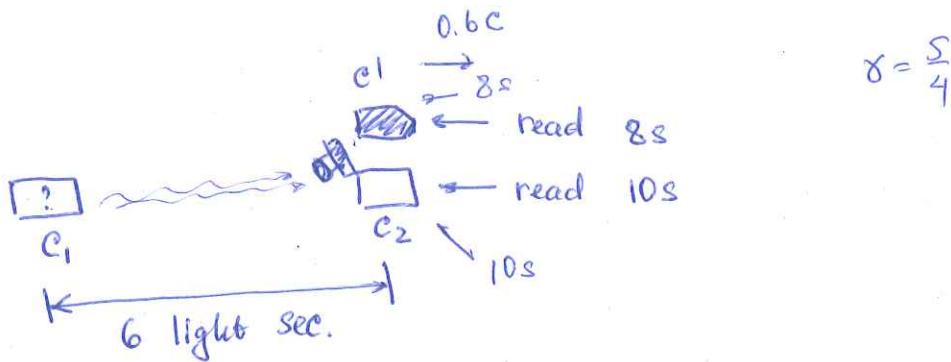


Follow-up question: on three-clock problem

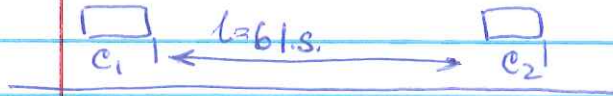
Let's imagine that at the instant Bob's shuttle passes the C2 clock, he turns around and take an instant photograph of the C1 clock (with his super-fancy future smartphone with Hubble-telescope-like optics).

What time the C1 clock would display on that picture?



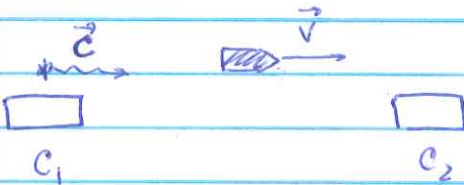
Alice's perspective

$t=0$

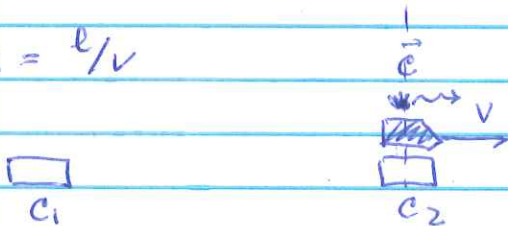


$t=t_A$

don't know



$t=10s = l/v$



Time travelled by the shuttle : $\frac{l}{v} = 10s$

Time travelled by light : $\frac{l}{c} = 6s$

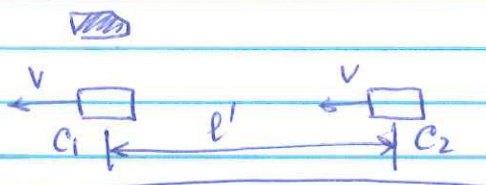
The pulse needs to depart $(10-6)=4s$ after the shuttle departs, so $t_A = 4s$

Bobs perspective

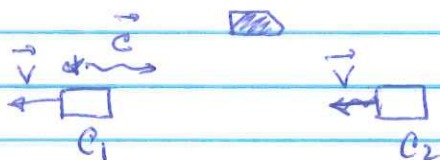
$$t' = 0$$

Length contraction

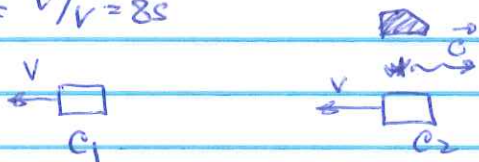
$$l' = \frac{l}{\gamma} = 4.8 \text{ s}$$



$$t'_B = ?$$



$$t = \frac{l'}{v} = 8 \text{ s}$$



Between t'_B and $t = \frac{l'}{v}$ the light pulse and the clock C_1 covered the distance l'

$$l' = \underbrace{c \left(\frac{l'}{v} - t'_B \right)}_{\text{distance travelled by the light}} + \underbrace{v \left(\frac{l'}{v} - t'_B \right)}_{\text{distance travelled by the shuttle}}$$

$$l' = \frac{c}{v} l' - ct'_B + l' - vt'_B \Rightarrow t'_B = \frac{c}{v} \frac{l'}{c+v}$$

$$t'_B = \frac{1}{0.6} \frac{4.8}{1.6} = 5 \text{ s}$$

Thus, the clock C_1 travelled for 5s (in BRF) before emitting the light. However, as moving clocks they run slower, so we need to account for the time dilation, so in C_1 RF only $5\text{s}/\gamma = 4\text{s}$ have passed, so the image carried by light will read 4s

Picture - perfect problem
(back to our three clocks)

click!
8:00 C_1

C_1

10:00 C_2

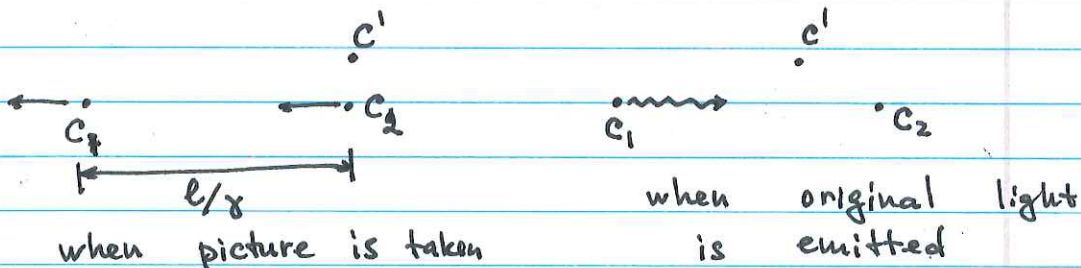
In order to be captured by the camera at C_2 location at $t=10s / t'=8s$, the light from C_1 have to travel there from C_1 location.

Scenario 1: It takes 6 seconds for light to travel b/w C_1 and C_2 , so
2.00 if Bob's clock read 8s, then the original reading was $8s - 6s = 2s$.

Wrong, since at Bob's RF distance is shorter

Scenario 2: In Bob's RF the distance b/w two clocks is contracted $\frac{4}{5} \cdot 6c \cdot s$, so
3.20 it takes light $0.8 \cdot 6s = 4.8s$ to travel so the picture must show $8s - 4.8s = 3.2s$

Wrong, since in Bob's RF the clocks move, so the light have to travel a shorter distance



During the light travel time st' , C_1 move by $v \cdot st'$ and light travels by $c \cdot st'$, together covering the distance b/w the clocks

$$c\Delta t' + v\Delta t' = \ell/x \quad \Rightarrow \quad \Delta t' = \frac{\ell/x}{c+v} = \frac{4.8\text{s}}{1.6} = 3\text{s}$$

Scenario 3: $8\text{s} - 3\text{s} = 5\text{s}$

5.00 Wrong, because C_1 show proper time in Alice's RF, and her time is dilated. It is true, that it takes light 3s in Bob's RF to reach the camera, so the shuttle travels for 5s before the light is emitted. But in Alice's RF (moving w/respect to Bob), it corresponds only to $\frac{4}{5} \cdot 5\text{s} = 4\text{s}$ time passing. Thus C_1 clock will display 4s when light leaves ~~them~~ it to catch up with Bob's camera at C_2 rendezvous point

Short cut: the picture must be identical if taken by Bob or Alice, if they do it at the same time at the same location. And the problem is much more straightforward for Alice: ~~her~~ since the light takes 6s to travel b/w the two clocks, it must have left C_1 at $10\text{s} - 6\text{s} = \underline{\underline{4\text{s}}}$